EE 591 K. Nayak
MR Imaging and Reconstruction

Homework #4

Due: Monday, January 27th (in class)

- This homework assignment depends heavily on Matlab. You will write simple image reconstruction scripts and a simple Bloch simulator, and then simulate excitation pulses as well as some off-resonance artifacts.
- You may work on the homework in groups, but must each submit independently and provide your own explanations of the images. If you are not an experienced Matlab user, please work with someone who is.

1. **Cartesian Image Reconstruction**

   Image reconstruction from cartesian k-space samples (as in 2DFT) primarily depends on an FFT.

   Generally in MR image reconstruction, the origin for both k-space data and the reconstructed image is at the center of the image or data array. However, the FFT operates using a convention where the origin is at the beginning of the array, or at the corner of the 2D array. To do a centered FFT or inverse FFT, we use functions like the following, which incorporate an fftshift().

```
% function im = ift(dat)
%
% Function does a centered inverse 2DFT of the data:

function im = ift(dat)

im = fftshift(ifft2(fftshift(dat)));
```

   Load the head dataset `headraw.mat` and reconstruct the head image:

```
> load headraw.mat
> im = ift(headraw);
> dispimage(abs(im));
```

   Submit code, images, and answers for the following questions:

(a) Upsample the original raw data matrix to 512x512 by inserting lines of zeros in between the data samples (in both directions). What happens to the reconstructed image? Anything useful?

(b) Zero-pad the original raw data matrix up to 512x512 and reconstruct. What happens to the reconstructed image? Anything useful?

2. **Test Objects**

When simulating sequences it's often useful to fabricate data for hypothetical objects. An easy way to do this is to design a test object that is made up of shapes with continuous Fourier transforms that are easy to describe analytically.

For example, a circular object rect(r) would have a continuous fourier transform that is jinc(k_r), and rect(x)rect(y) has a continuous fourier transform that is sinc(k_x)sinc(k_y).

Use the following commands to generate one possible test object:

```
> krange = (-128:127)/256;
> [ky kx] = meshgrid(krange,krange);
> kr = sqrt(kx.^2 + ky.^2);
> tx_circ = 25*25*jinc(25*kr);              % FTx of circle with diameter 25
> tx_rect = 5*30*sinc(5*kx).*sinc(30*ky);   % FTx of rectangle 5x30
> tx = tx_circ .* exp(-j*2*pi*kx*30) ...
>    + tx_rect .* exp(-j*2*pi*(-10*kx + 20*ky));
> im = ift(tx);
> figure;
> dispimage(abs(tx));
> figure;
> dispimage(abs(im));
```

(a) Examine the reconstructed object carefully (zoom into different areas, and window the image). Describe artifacts that you find, and why they are there.

(b) Generate a raw data matrix for your own test object. You may use the provided `rectangle_fourier()` and `ellipse_fourier()` functions. Be creative, and make sure the test objects use up a large portion of the field of view, and that they contain some elements that have high spatial resolution.

(c) When we simulate an object containing different tissues, it is beneficial to create raw data matrices for each tissue type. For example:

```
> tx_water = 100*100*jinc(100*kr) - 50*50*jinc(50*kr);
> tx_fat = 1.2 * (50*50*jinc(50*kr));
> tx = tx_water + tx_fat;
> im = ift(tx);
> dispimage(abs(im));
```

This test image is a circle of fat surrounded by a circle of water. Note that we made fat 20% brighter than the water.

With separated raw-data for the fat and water, it's easier to simulate the effects of off-resonance (or other tissue differences). In 2DFT imaging, the signal phase due to off-resonance can be expressed as a function of k_x. Suppose that the slice is excited at t = 0, and readouts occur from t = 10 to 30 ms. Express t as a function of k_x (Note that the k_x matrix values go from -0.5 to 0.5). Suppose we are imaging at 3 Tesla so that Δf = -440 Hz for fat. Apply the appropriate phase to the fat raw data, and reconstruct the fat and water image. What do you see?

You just reconstructed the image for a 20 ms readout with a 20 ms TE. Repeat the reconstruction for a 22 ms TE, 24 ms TE, and 28 ms TE. Do you notice any changes in the region of overlap? What is happening? *Hint: look at the image phase* `dispimage(angle(im))`.

Now reconstruct the image for a 8 ms readout with a 20 ms TE. What do you see?

(d) In PR and spiral imaging, the *time map* is a function primarily of k_r. Suppose that we are using single-sided PR sequence with readouts again occur from t = 10 to 30 ms (this time TE = 10 ms). If t is a linear function of k_r, reconstruct the resulting image. Remember to zero-out portions of the raw data matrix which are outside the circular region that is acquired (i.e. where $k_r > 0.5$).

Repeat the reconstructions if the readouts are shortened to 10 ms? and again shortened to 4 ms? (always starting 10 ms after the excitation)

Explain your findings.

3. **Bloch Simulation**

In this problem you will implement a Bloch equation simulator, and use it to test the performance of excitation pulses.

In the presence of RF pulses, Gradient fields, and relaxation, the Bloch equation dictates the behavior of spin magnetization. In the rotating frame, the magnetization vector experiences excitation, precession, and relaxation. In reality these three things are happening simultaneously, but a good approximation is to look at short time intervals (dt) and simulate the effects of the three separately.

If including relaxation, off-resonance, precession due to gradients, and excitation, you might use code similar to the following:

```
> Mo = 1;
> M = [0;0;Mo];       % equilibrium magnetization
...
> E1 = exp(-dt/T1);
> E2 = exp(-dt/T2);
> A = [E2 0 0; 0 E2 0; 0 0 E1];
> b = [0;0;(1-E1)];
```

```
...
> M = xrot(gamma*B1*dt) *M;      % excitation_x:  rotation about the x axis
> M = zrot(2*pi*df*dt) *M;       % precession due to off-resonance (df)
> M = zrot(gamma*(Gx*x+Gy*y+Gz*z)*dt) *M;
                                 % precession due to Gradients
> M = A*M+b*Mo;                  % relaxation
```

In this problem, you will be simulating excitation pulses, off-resonance, and gradients. Relaxation is ignored.

Browse the provided `simpulse.m` file and verify that it simulates the rotations experienced by a spin at position `z` due to a given `b1` (RF pulse in Gauss), `gz` (Z gradient in Gauss/cm), and `df` (off-resonant frequency in Hz).

For the following two parts assume that magnetization begins at equilibrium, that Mo = 1, and ignore relaxation. The waveforms you will load have a sampling rate of 4μs. And gamma is 26752 rad/s/G.

(a) Load the slice selective RF pulse `sliceselect.mat` which contains an RF pulse and associated Z-gradient. Simulate the excitation profile for spins with z-positions from -1 cm to 1 cm in 0.2 mm (or smaller) increments. Plot Mx, My, and Mz as a function of z position.

Repeat the simulation assuming off-resonance with Δf = -440 Hz (fat). What happens to the slice profile? What are the implications if we use this excitation pulse to image a slice in the body that contains both water and fat?

(b) Load the spectral-spatial RF pulse `specspat.mat` which also contains an RF pulse and associated Z-gradient. This is an interesting pulse design that excites a limited range of frequencies.

Plot Mx, My, and Mz as a function of z position (for $\Delta f = 0$).

Plot Mz, My, and Mz as a function of off-resonance (for z = 0). Let Δf range from -1 kHz to 1 kHz in 25 Hz increments.

OPTIONAL PART (if time permits): Make an image of the resulting transverse magnetization ($|M_r| = |M_x + iM_y|$), for locations ranging from z = -1 cm to 1 cm, and Δf ranging from -1 kHz to 1 kHz in 25 Hz increments.

(c) Modify `simpulse.m` to incorporate relaxation. Assume T1 = 100 ms. Consider the slice selective pulse in part (a) and determine how small T2 needs to be before it noticeably affects the excitation profile.

Explain the slice profile changes that you notice.