

# Fast Gradient Echo Sequences Including Balanced SSFP

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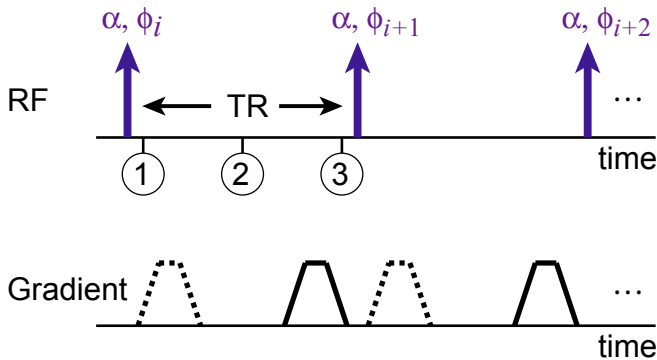
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## 1 Introduction

Many common magnetic resonance imaging sequences are classified either as spin echo or gradient echo techniques. Generally, a gradient echo occurs when spins are coherent regardless of their position within the sample. This occurs whenever the net gradient area since a previous echo is zero. In a spin echo, spins are coherent regardless of their resonance frequency [1]. Here we will focus on gradient echoes in more detail, including how they evolve, and the so-called gradient-echo pulse sequences. These include gradient spoiled sequences, RF-spoiled sequences and balanced steady-state free precession (SSFP) sequences.

### 1.1 General Rapid Gradient Echo Sequences

Rapid gradient echo sequences consist of a single radio frequency (RF) excitation, imaging gradients and acquisition, and spoiler gradients. Figure 1 shows a basic example of the RF and spoiling gradients. The RF pulse has some flip angle  $\alpha$  and some phase angle,  $\phi$ . For simplicity, we omit imaging and slice-select gradients, and assume that the spoiler gradients are along only one axis.



**Figure 1:** Gradient echo sequences generally consist of repeated RF pulses, and some type of spoiling gradients. The RF flip angle ( $\alpha$ ) is usually constant, while the phase angle ( $\phi$ ) can be constant, or can increment linearly or quadratically with the repetition number. Spoiling gradients usually occur at the end of the repetition (solid lines), but may also occur at the beginning (dotted lines), or not at all. Three time points of interest, numbered 1-3, are used in analyzing the signal levels.

Four basic gradient-echo sequences can be derived from Fig. 1. *Balanced SSFP* (steady state free precession) sequences have no spoiler gradients. *Gradient-spoiled sequences* have a spoiler at the end of the repetition (Fig. 1, solid gradient.). *Reversed gradient-spoiled* sequences have a spoiler gradient immediately after the RF pulse, before imaging. (Fig. 1, dotted line). Finally, *RF-spoiled* sequences use a gradient spoiler at the end of the repetition and an additional quadratic phase increment for  $\phi$ . By first examining the dynamics of balanced SSFP, we can build up the other three sequences.

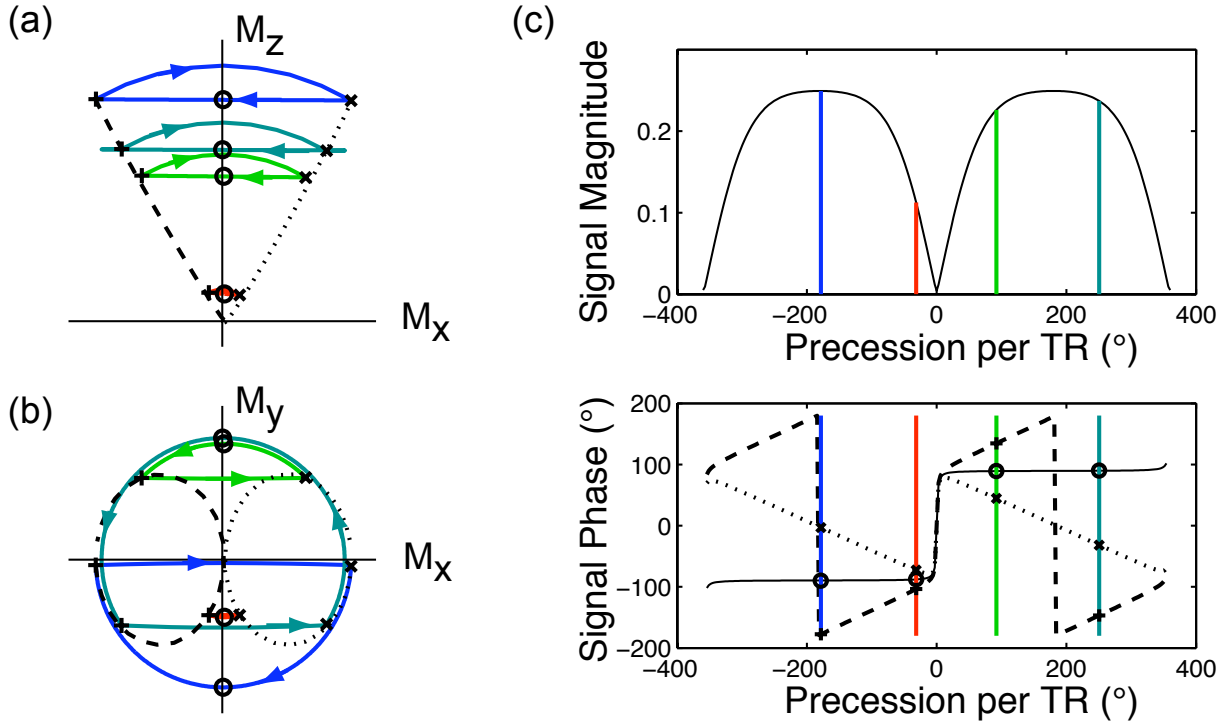
### 1.2 Spin Dynamics

The dynamics of spins in an MRI system with radio frequency (RF) and gradient coils are accurately described by the Bloch equation. Given RF and gradient waveforms, as well as the resonance frequency, relaxation times and position of a spin, the (3x1) magnetization vector can be easily modeled using a matrix formalism [2], where an RF pulse is represented by a series of rotation matrices about an arbitrary axis in the transverse ( $M_x - M_y$ ) plane. Precession, due to the resonance frequency as well as the effect of gradients is also represented by a series of rotation matrices about the longitudinal (z) axis. Relaxation is represented as a matrix multiplication, with an addition of a recovery term. Using a time step that is small compared with the rate of any of these processes, these three matrices can be applied sequentially and still provide a very good model of the spin dynamics.

### 1.3 Magnetization Steady States

When the same sequence of RF rotations, precession and relaxation is repeated, a steady state forms, where the magnetization at some point in the sequence is the same from one repetition to the next. There are three cases:

- $TR > 2T_1$  and  $TR > 2T_2$ . Here the magnetization at the start of the sequence relaxes completely to the equilibrium magnetization,  $M_0$ . The “steady state” is just the equilibrium magnetization.
- $TR > 2T_2$  but  $TR < 2T_1$ . Here all transverse magnetization will be zero at the start of the sequence. However, the longitudinal magnetization reaches a steady state.



**Figure 2:** Magnetization dynamics in balanced SSFP. (a) and (b) show the path of magnetization in the  $M_x - M_z$  and  $M_x - M_y$  planes, while (c) shows the signal ( $M_{xy}$ -magnetization) magnitude and phase as a function of resonance frequency. Colored lines represent single resonance frequencies. The  $\times$ ,  $\circ$  and  $+$  symbols represent magnetization at time points 1, 2 and 3 in Fig. 1. Immediately after the RF pulse, steady-state magnetization is distributed along an ellipse ( $\times$  symbols and dotted black line in a,b). Spins precession over  $TR$  (colored lines in b), aligning along the  $M_y$  axis at  $TR/2$  ( $\circ$  symbols). Immediately before the RF pulse, magnetization is again distributed along an ellipse ( $+$  symbols and dashed black line).

- $TR < 2T_1$  and  $TR < 2T_2$ . Both longitudinal and transverse magnetization reach a non-zero steady state.

We will concentrate on the third of these cases, which applies to most rapid gradient echo sequences.

## 2 Balanced SSFP Dynamics

Balanced SSFP consists simply of RF pulses that are repeated every  $TR$ , with no spoiling gradients [3]. The dynamics of other rapid gradient echo sequences can be derived from those of balanced SSFP. First we assume a basic sequence with the following dynamics:

- RF rotation of an angle  $\alpha$  about the  $y$  axis.
- Precession  $\theta$  due to resonance frequency offset.
- $T_1$  and  $T_2$  relaxation.

After numerous repetitions, the magnetization reaches a steady state, that can be calculated analytically by propagating the above dynamics through a sequence repetition [4–7]. However, it is important to try to intuitively understand the dynamics [8–10]. For moderately large  $\alpha$ , there are two cases of interest (shown in Fig. 2):

1. When the amount of precession is very small, ( $\theta$  close to  $0^\circ$ ), the magnetization mostly rotates around the  $y$  axis. A steady state forms the effects of relaxation and RF rotation perfectly cancel, which, for large  $\alpha$  requires the magnetization length to be very small (Fig 2, red line).
2. When precession is significant, it almost completely cancels RF nutation. The magnetization follows the path(s) shown in Fig. 2. The signal is refocused midway through the sequence along either the positive or negative  $y$ -axis, depending on the direction and amount of precession.

The magnetization length is such that the direction of relaxation is orthogonal to the magnetization vec-

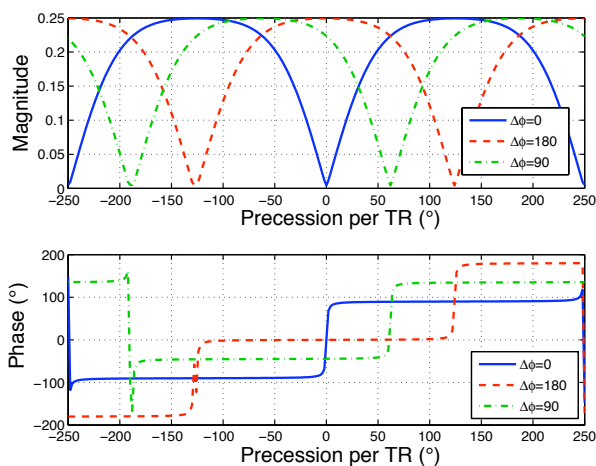
tor. This means that as the magnetization vector direction approaches the  $M_x - M_y$  plane, the vector length becomes smaller.

The motion in the steady state is a function of several parameters. First, the RF flip angle,  $\alpha$ , and phase,  $\phi$ , determine the direction and amount of rotation. The resonance frequency and repetition time,  $TR$ , give the amount of precession between RF pulses. The relaxation times,  $T_1$  and  $T_2$ , together with  $TR$  determine the amount of magnetization decay and recovery.

The signal magnitude varies periodically with resonant precession with a period of  $360^\circ$ . Immediately before, and immediately after the RF pulse, the transverse magnetization component is confined to a half plane. The signal is also *refocused* by the RF pulse [11], resulting in piecewise constant phase midway between RF pulses.

## 2.1 RF Phase Cycling

If the RF and receiver phase are incremented by  $\Delta\phi$  on each repetition, the magnetization dynamics of Fig. 2 will be altered. A constant phase increment can be viewed as a shift in the center frequency of  $\Delta\phi/TR$  [4, 12]. The result is that the signal profile in Fig. 2c is shifted along the precession axis by  $\Delta\phi$ , and an additional phase of  $\Delta\phi/2$  is added to the signal phase.



**Figure 3:** Balanced SSFP signal magnitude and phase as a function of resonant precession ( $^\circ$ ) for  $\Delta\phi = 0^\circ$ ,  $\Delta\phi = 180^\circ$ , and  $\Delta\phi = 90^\circ$ . The magnitude and phase profiles shift along the precession axis by  $\Delta\phi$ , and the phase changes by  $\Delta\phi/2$ .

## 2.2 Frequency Modulation

The RF and receiver phase can also be modulated using *quadratic phase* increments, where  $\Delta\phi$  is increasing by a constant factor on each repetition. This results in the center frequency shifting with time, so the signal profile of Fig. 2 slowly shifts along the horizontal axis. If the quadratic phase increment is small, there is little distortion to the signal profile, and spins can “track” the frequency change [13]. For larger phase increments, the profile becomes somewhat distorted. Either way, spins will form a periodic steady state with a period equal to the time for the frequency shift to equal  $1/TR$ .

## 3 Pulse Sequences

The previous section described the steady-state that forms when a periodic sequence of RF rotations, precession, and decay are applied. We now describe several different pulse sequences using these concepts to understand the signal formation with each sequence. In all cases, we use the x, y and z axes as the readout, phase-encode and slice-select axes respectively.

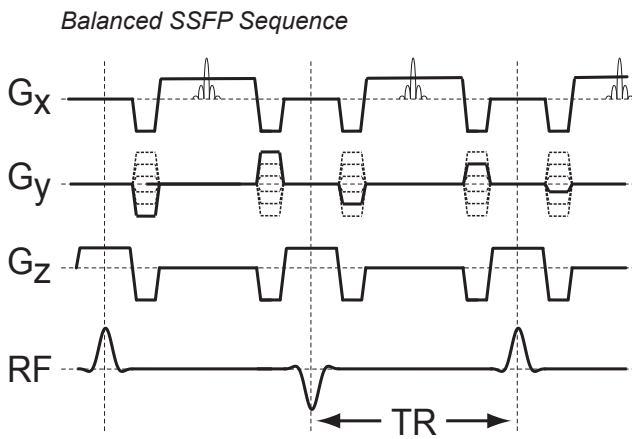
### 3.1 Balanced SSFP

Balanced SSFP sequences (TrueFISP, FIESTA, Balanced-FFE, True SSFP) have the signal characteristics described already. Normally, the RF pulse sign alternates, which is equivalent to a phase increment of  $180^\circ$ . This shifts the signal profile so that “on-resonant” spins produce high signal (Fig. 3, dashed line). Figure 4 shows a balanced SSFP pulse sequence. Note that all gradient waveforms are balanced, that is they have zero net area over a full repetition.

Balanced SSFP sequences give the highest signal level of the rapid gradient echo sequences [14, 15]. The signal shape is highly sensitive to resonance frequency so minimizing precession, and thus  $TR$ , is important for avoiding “banding artifacts” or signal variations across images [16]. However, the minimum time required to achieve adequate spatial resolution or to limit RF power absorption limit the minimum  $TR$  that can be achieved. Furthermore, susceptibility variations make balanced SSFP more difficult at higher field strengths.

### 3.2 Gradient-Spoiled Echo

The gradient-spoiled sequence, (FE, GRASS, FISP, FAST), uses a spoiler gradient *at the end* of the sequence



**Figure 4:** Balanced SSFP pulse sequence. All gradient waveforms are fully rewound, or balanced. The RF pulse sign usually alternates, so that a high signal is produced for on-resonant spins. The signal echo location is shown on the  $G_x$  waveform.

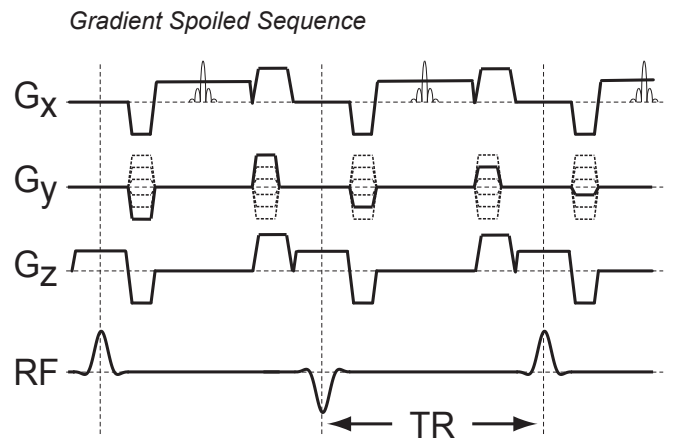
as shown in Fig. 5 [7, 17]. Each spin now sees a position-dependent rotation in addition to that due to free precession. Note, however that the steady state for each spin at time points 1 and 3 (Fig. 1) is the same as that for balanced SSFP, when the gradient-induced rotation is included in the precession per  $TR$ .

The spoiler gradient area is chosen to induce rotations that range over at least  $360^\circ$  within each voxel, and it is assumed that there is a uniform distribution of precession values within a voxel. Thus the signal immediately after the RF pulse is the average transverse component of all precession values, i.e. the average of the balanced SSFP signal on the dotted ellipse in Fig 2b. Between the RF pulse and the echo time, some dephasing of the signal due to frequency variations can occur. This results in  $T_2^*$  signal loss.

There are many variations of gradient-spoiled echo sequences, depending on the direction and size of the spoiler gradient. However, if the assumption can be made that the unbalanced gradient induces numerous rotations within a single voxel, the signal is similar for all variations.

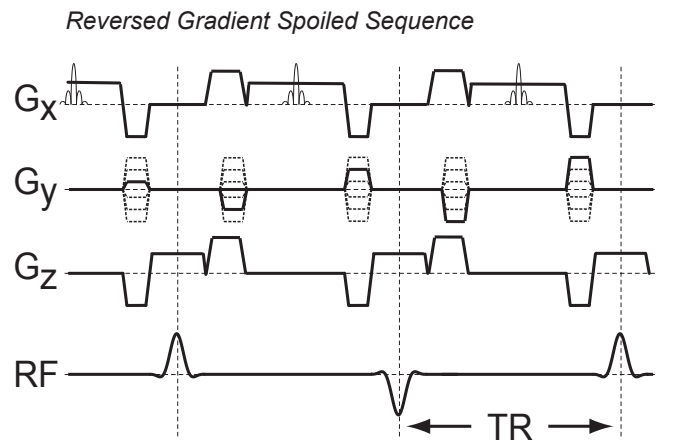
### 3.3 Reversed Gradient-Spoiled Echo

The reversed gradient-spoiled echo sequence (CE-FAST, SSFP, T2-FFE and PSIF) instead uses a gradient spoiler at the beginning of the sequence, as shown in Fig. 6 [17]. As with the gradient-spoiled sequence, the steady state magnetization for any single spin is the same as for balanced SSFP when the precession due to the gradient spoiler is included. However, imaging occurs after the



**Figure 5:** Gradient spoiled pulse sequence. In this example, a spoiler gradient is included on both the readout (x) and slice-select (z) axes. The signal echo location is shown on the  $G_x$  waveform.

spoiler gradient, at time point 3 in Fig. 1. The signal is the average transverse component of the balanced SSFP magnetization, or the average signal on the dashed ellipse in Fig 2b. Again,  $T_2^*$  signal loss occurs, but should be calculated in reverse from time point 3 to the point of the imaging echo.

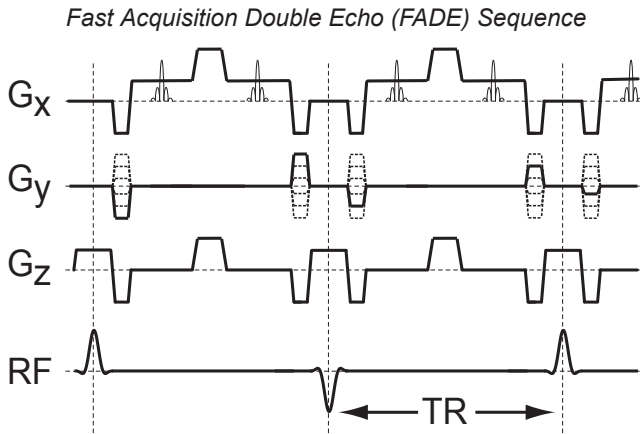


**Figure 6:** Reversed gradient spoiled pulse sequence. This is precisely the reverse of the gradient spoiled sequence. Note that it is important to “rephase” the readout gradient and “defocus” the slice-select gradient.

### 3.4 Fast Acquisition Double Echo (FADE)

The fast acquisition double echo (FADE) sequence acquires both the gradient-spoiled echo and the reversed gradient-spoiled echo in a single repetition [18]. This is achieved by placing the gradient spoiling in the middle of

the sequence as shown in Fig. 7. The echo characteristics are unchanged from the descriptions in §3.2 and §3.3.



**Figure 7:** FADE pulse sequence, which acquires both the gradient-spoiled echo and the reversed gradient-spoiled echo (Shown on  $G_x$  waveform). A spoiler gradient is included on both the readout (x) and slice-select (z) axes, midway between RF pulses.

### 3.5 RF Spoiling

Gradient spoiled, reversed gradient spoiled and balanced SSFP sequences all have a signal with “ $T_2/T_1$ ” contrast. The  $T_2$  dependence can be virtually eliminated by RF spoiling [19–21]. RF spoiling quadratically increments the phase of the RF pulse with the result that residual transverse magnetization before the RF pulse is not refocused and can be neglected. The sequence looks exactly like the gradient-spoiled sequence (Fig. 5). The signal in RF spoiled sequences (SPGR, FLASH, T1-FFE) can be calculated by numerical simulation, but is well approximated by simply neglecting residual transverse magnetization prior to each RF pulse (case 2 described in §1.3).

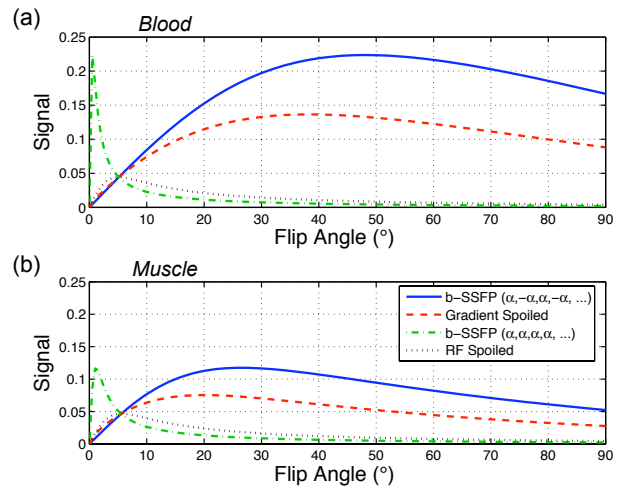
## 4 Imaging Considerations

In this section, we briefly discuss the signal considerations with the rapid gradient echo pulse sequences, including relative signals and contrast of different sequences.

### 4.1 Signals and Contrast

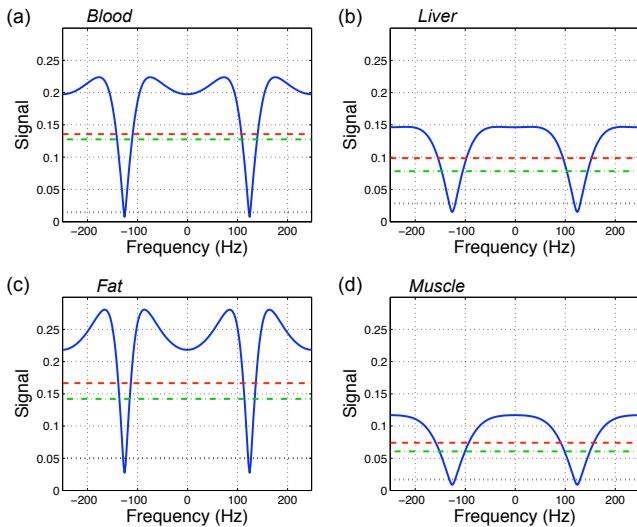
The contrast behavior of rapid gradient-echo sequences is highly dependent on flip angle.  $TR$  is usually kept as

short as possible to minimize scan time or to reduce banding artifacts in balanced SSFP. Figure 8 shows the flip-angle dependence of four sequences: balanced SSFP with and without alternating the RF pulse sign, gradient spoiled sequences, and RF spoiled sequences. For certain  $T_1$  and  $T_2$  times, each sequence has an optimal flip angle that maximizes SNR. An interesting point to note is that at the Ernst angle,  $\cos^{-1}(e^{-TR/T_1})$ , which maximizes signal for RF-spoiled sequences, the signal is identical for all sequences. [5]. The flip angle is usually chosen to maximize a combination of SNR as well as contrast between the tissues of interest.



**Figure 8:** Signal as a function of flip angle for (a) blood and (b) muscle. Shown are the signals from balanced SSFP (0 Hz, alternating RF sign), gradient spoiled, balanced SSFP (0 Hz, non-alternating RF sign), and RF spoiled sequences. The Ernst angle,  $\cos^{-1}(e^{-TR/T_1})$ , signal for all sequences, and the peak signal for RF spoiled sequences [5]

Figure 9 compares the signal in balanced SSFP, gradient-spoiled, reversed gradient-spoiled and RF-spoiled sequences with identical  $TR$  and flip angle. Four tissues are compared: blood, liver, fat and muscle. The balanced SSFP signal is very sensitive to flip angle, and the shape also depends on the  $T_1/T_2$  ratio. At low  $T_1/T_2$ , the signal is “M-shaped” (fat, blood). At higher  $T_1/T_2$  (muscle, liver) or higher flip angles (not shown) the signal shape is more smooth. Balanced SSFP gives the highest signal. Gradient-spoiled and reversed gradient-spoiled sequences give similar signal levels, the latter being more  $T_2$ -weighted. Finally, RF-spoiled sequences are purely  $T_1$ -weighted, and give the lowest signal of these four sequences.



**Figure 9:** Comparison of signals in different tissue with balanced SSFP, gradient spoiled, reversed gradient spoiled, and RF spoiled sequences. (a) Blood,  $T_1/T_2 = 1000/200$  ms. (b) Liver,  $T_1/T_2 = 500/43$  ms. (c) Fat,  $T_1/T_2 = 270/85$  ms. (d) Muscle,  $T_1/T_2 = 850/47$  ms. Other sequence parameters are  $TR=4$  ms, flip angle= $30^\circ$ .  $TE=0$  ms for gradient spoiled and RF spoiled,  $TE=2$  ms for balanced SSFP, and  $TE=TR$  for reversed gradient echo. Note that the balanced SSFP signal is highest, but also is sensitive to frequency variations. The reversed gradient spoiled sequence produces more  $T_2$  weighting than the gradient spoiled sequence. Finally, RF spoiled sequences produce the lowest signal, but it is purely  $T_1$ -weighted.

## 4.2 Signal-to-Noise Ratio

The signal-to-noise ratio (SNR) is one of the most important considerations in selecting imaging parameters for MRI. SNR is proportional to voxel size, the square root of the total acquisition time, and the signal level for the given sequence and parameters.

The sequences shown here typically achieve good SNR by using a short  $TR$  so that the signal is averaged over many repetitions, often with 3D phase-encoding. However, as  $TR$  is shortened, a greater proportion of time is taken by RF pulses and preparatory gradients such as phase-encoding, dephasing and rephasing. This means that the “acquisition duty cycle” or the portion of  $TR$  spent acquiring data is critical. Maximizing SNR becomes a trade-off between keeping  $TR$  short to minimize  $T_2^*$  loss or banding artifacts (in balanced SSFP) and keeping the readout duty cycle long.

## 5 Summary

Rapid gradient echo sequences with very short repetition times are commonly used in MRI. The signal behavior of these sequences depends strongly on the residual magnetization prior to each excitation, which can be altered by using combinations of gradient spoiling and RF spoiling. After many sequence repetitions, the magnetization reaches a steady-state, where it varies periodically. The steady state calculation for the fully-balanced SSFP sequence can be used to understand the signal formation in gradient spoiled and RF spoiled sequences. Imaging parameters must then be chosen to optimize SNR and contrast for specific applications.

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