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On the UNFOLD Method

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A graphical formalism is presented, showing that "UNaliasing by Fourier-encoding the Overlaps Using the temporaL Dimension" (UNFOLD) is equivalent to sampling *k-t*-space in a sheared grid pattern. Discrete regular sampling in *k-t*-space leads to periodic replication of the support region in *x-f*-space. Thus, the maximum acceleration achievable by UNFOLD is equivalent to the maximum packing of support regions in *x-f*-space. When the support region is separable along the *x* and *f* axes, the reconstruction can be performed separately for each *k*. UNFOLD can be combined with SiMultaneous Acquisition of Spatial Harmonics (SMASH) to further accelerate acquisition. However, a straightforward combination of the methods has been shown to result in a size restriction, which limits the portion of the field of view (FOV) with a larger temporal bandwidth to only a quarter of the FOV. Two solutions are presented to overcome this restriction. Magn Reson Med 47:202–207, 2002. © 2002 Wiley-Liss, Inc.

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Recently, Madore et al. (1–3) proposed the "UNaliasing by Fourier-encoding the Overlaps using the temporaL Dimension" (UNFOLD) method for dynamic imaging. This method speeds up data acquisition by sampling only a fraction of *k-t*-space. Compared to parallel acquisition methods (4,5) for fast imaging, it has the advantage of not requiring special hardware, such as phased-array coils. Since its introduction, UNFOLD has attracted the attention of other researchers as well (6–10). However, from the published descriptions of UNFOLD (1–3), it is difficult to ascertain the capability of the method, such as the maximum achievable acceleration factor. The purpose of this work is to present a theoretical analysis of UNFOLD, demonstrating that the method admits a simpler interpretation, which reveals its maximum capability and new ways of applying it (e.g., combined with parallel imaging) in an intuitive and geometric manner. Additionally, the new interpretation highlights the relationship between UNFOLD and other existing methods.

THEORY

UNFOLD Cardiac Imaging

UNFOLD has been applied to cardiac imaging to achieve a twofold reduction in data acquisition (1–3). In that application, it assumes that half of the FOV (e.g., the portion over the heart) contains a much wider range of temporal frequencies (i.e., a larger temporal bandwidth) than the remaining FOV (e.g., adjacent portions in the chest). This is illustrated in Fig. 1a, in which the shaded cross shape indicates the support region containing most of the signal energy, x denotes the spatial position along the phase-encoding direction, and f denotes the temporal frequency. The readout direction is omitted for simplification, and it is frequency-encoded as in conventional Fourier imaging. In this figure, the "more dynamic" region (i.e., the rectangular region with a larger temporal bandwidth) is shown at the center of the x-axis for illustration purposes only. In general, it can be shifted to any position along x. Moreover, it can be shifted to a different x for every image column along the readout direction. In other words, its position can be adapted on a column-by-column basis to follow the anatomy more closely.

Data acquisition speed is doubled by acquiring either the odd or even phase-encode lines at alternate time frames. In the original UNFOLD publications (1–3), this acquisition scheme is described in a frame-by-frame manner as follows. At each time frame, k-space is undersampled by a factor of 2. This leads to a one-half reduction of the FOV and a subsequent twofold aliasing of the image contents. Since the sampling pattern is shifted at every other time frame, the aliased portion of the image contents alternates between positive and negative signs at successive time frames. As a result, the aliased portion can be separated from the unaliased portion by high-pass filtering along the temporal axis.

Instead of this frame-by-frame analysis (1–3), the present work shows that the acquisition scheme can be analyzed in a more straightforward and intuitive manner, directly in *k-t*-space. Figure 1b shows the acquisition scheme in *k-t*-space, which can be viewed as sampling *k-t*-space in a sheared grid pattern (Fig. 1c). From the properties of the Fourier transform, discrete regular sampling in *k-t*-space leads to periodic replication in the reciprocal space (*x-f*-space). The replicates are arranged in a sheared grid pattern (Fig. 1d) according to the point spread function (Fig. 1e), which is the inverse Fourier transform of the sampling grid in *k-t*-space. The replicates do not overlap because of the shape of the support region. Thus, an unaliased copy of the signal can be recovered.

This graphical formalism (Fig. 1a–e) suggests a straightforward alternative to the original reconstruction algorithm described by Madore et al. (1–3), involving four steps: 1) zero-fill the unacquired phase-encode lines; 2) apply the inverse Fourier transform along the spatial and temporal directions; 3) set all signals outside the support region in *x-f*-space to zero (this is equivalent to the filtering in UNFOLD); and 4) apply the Fourier transform along the temporal direction to obtain images in *x-t*-space.

The main difference between this algorithm and the original UNFOLD algorithm is that the aliasing is directly resolved through steps 1 and 2 (i.e., zero-filled Fourier

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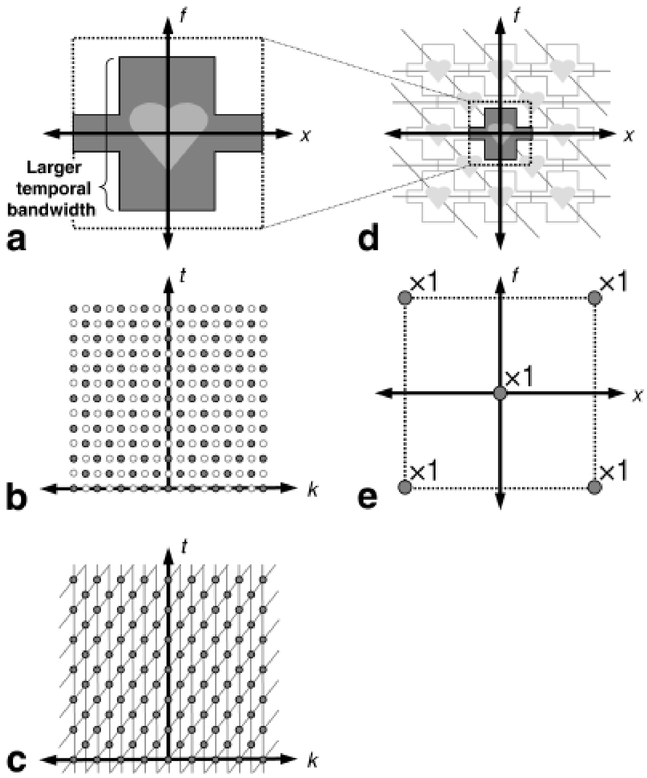

FIG. 1. UNFOLD for cardiac application with Cartesian sampling in *k*-space. **a:** Shaded cross shape indicates the support region in *x-f*-space (*x*: spatial position along phase-encoding direction, *f*: temporal frequency). The signal intensity is assumed to be negligible outside the support region. The heart shape identifies the portion of *x* with a larger temporal bandwidth. **b:** The *k-t*-space sampling pattern with gray (●) and white dots (○) indicating sampled and skipped positions, respectively (*k*: *k*-space position along phase-encoding direction; *t*: time). The sampling pattern is equivalent to a sheared grid pattern shown in **c**. A twofold acceleration is achieved with this sampling pattern. **d:** Periodic replication in *x-f*-space. The size scale is reduced compared to **a**. **e:** The point spread function determining the replication pattern in **d**. Values in **e** indicate the relative weights of the point spread function.

reconstruction along spatial and temporal directions). More importantly, this algorithm elucidates the mechanism by which UNFOLD resolves the aliasing from the undersampled *k*-space. This is accomplished by judiciously packing the replicates of the support region in *x-f*-space in order to avoid overlap. Figure 1d shows that for the present support region, the replicates are already packed together as tightly as possible when the data acquisition is doubled. Therefore, it can be seen geometrically that the acceleration factor cannot increase beyond 2 without overlapping the replicates, which will result in aliasing (11).

UNFOLD fMRI

In the case of fMRI (1–3), the support region has a different shape, as shown in Fig. 2a. The support region consists of horizontal strips centered around the zero frequency (DC) and the harmonics of the stimulation paradigm. In Fig. 2b and c, a fourfold acceleration is achieved by sampling every fourth phase-encode line. The resulting periodic replication of the support region and the point spread function are shown in Fig. 2d and e, respectively. As before, an unaliased copy of the signal can be recovered by setting all signals outside the support region to zero.

The support region in Fig. 2a has the special property of being separable along the *x* and *f* directions. Thus, reconstruction can be performed separately for each *k*, thereby simplifying the computation significantly. In this case, UNFOLD is similar to the dynamic imaging by motion estimation (DIME) method (12).

Madore et al. (1–3) chose to use spiral sampling of *k*-space in their fMRI example, which leads to several important differences from the Cartesian sampling case. First, *k* refers to the index of the spiral interleave, and not the *k*-space position along the phase-encoding direction. Secondly, *x* refers to the reciprocal axis of *k*, but it no longer corresponds to a spatial axis because the *k*-space trajectory

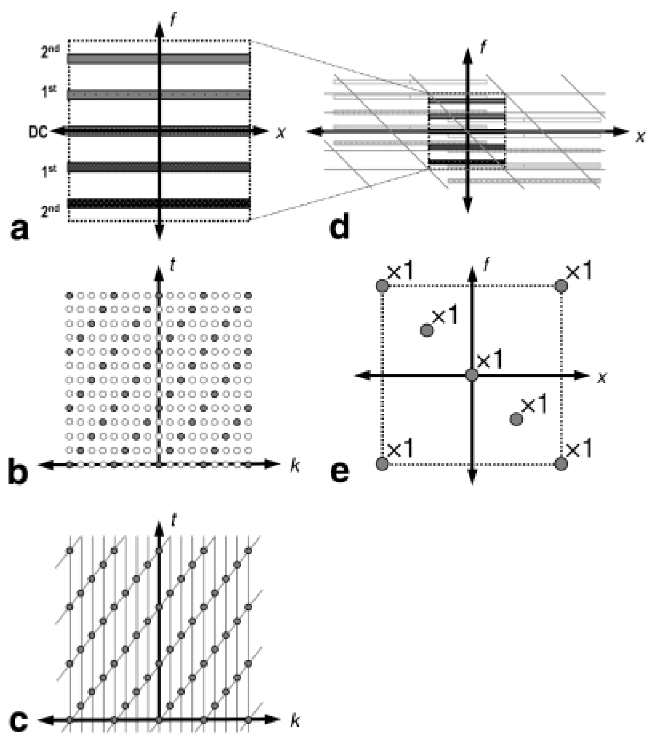

FIG. 2. UNFOLD for fMRI application with spiral sampling in *k*-space. **a:** Shaded horizontal stripes indicate the support region in *x-f*-space (*x*: reciprocal axis to *k* in **b**; *f*: temporal frequency). The horizontal strips are centered around the zero frequency (DC) and the first (1st) and second (2nd) harmonics of the stimulation paradigm. The signal intensity is assumed to be negligible outside the support region. The different shadings within the support region are for distinguishing the horizontal strips only. **b:** The *k-t*-space sampling pattern with gray (●) and white dots (○) indicating sampled and skipped positions, respectively (*k*: index of spiral interleave, *t*: time). The sampling pattern is equivalent to the sheared grid pattern shown in **c**. A fourfold acceleration is achieved with this sampling pattern. **d:** Periodic replication in *x-f*-space. Only five replicates are shown, to minimize clutter. The size scale is reduced compared to **a**. **e:** The point spread function determining the replication pattern in **d**. Values in **e** indicate the relative weights of the point spread function.

is now curved. As a consequence, the support region (Fig. 2a) is not localized along the *x* direction, but is spread to the full extent. If Cartesian sampling is used instead, it may be possible to confine the support region to a smaller area in *x-f*-space, thereby allowing an even tighter packing, as mentioned briefly by Madore et al. (3) In general, the problem of packing support regions has been analyzed in a lattice-theoretic approach by Willis and Bresler (13,14). In that work, Willis and Bresler (13,14) further considered the time-sequential-sampling constraint (i.e., only one data point can be acquired at a time), thus abolishing the implicit assumption that a set of phase-encode lines can be acquired instantaneously at each time frame.

Combining UNFOLD With SMASH

Over the last few years, considerable interest in fast imaging has focused on parallel imaging methods, particularly on SiMultaneous Acquisition of Spatial Harmonics (SMASH) (4), and SENSitivity Encoding (SENSE) (5). Thus, a natural question is whether UNFOLD can be combined with parallel imaging to further accelerate imaging speed (9,15). A preliminary analysis was presented by Kellman and McVeigh (9), who considered combining UNFOLD with parallel imaging to provide a twofold acceleration each, thus achieving a fourfold acceleration overall. They concluded that the combined method would work if the more dynamic portion of the FOV (i.e., the portion with larger temporal bandwidth) was restricted to one-quarter of the FOV only. In this section, a similar analysis of combining UNFOLD with SMASH is presented using the graphical formalism, which confirms the analysis of Kellman and McVeigh (9). Furthermore, it reveals two solutions to overcome the quarter-FOV restriction. Very recently, a third solution for overcoming this restriction was also presented by Madore (15). Since UNFOLD and parallel imaging are linear methods, these solutions are fundamentally related. They simply represent different practical approaches for solving the same inverse problem. The present work focuses on modifications of the *k-t*-space sampling pattern, while Madore's solution (15) relies on the spatial localization ability of phased-array coils.

In the present analysis, the support region is assumed to have a cross shape, with one-quarter of the FOV being more dynamic (Fig. 3a). Data acquisition speed is quadrupled by acquiring only every fourth phase-encode line in *k*-space, k_l (with l ranging from one to the number of phase-encode lines acquired at each time frame). The use of SMASH allows reconstruction of the *k*-space data in the adjacent phase-encode lines ($k_l + 1$) (4). At each time frame, the sampling pattern is shifted by two phase-encode lines. The overall sampling pattern in *k-t*-space is shown in Fig. 3b. The resulting periodic replication of the support region and the point spread function are shown in Fig. 3c and d, respectively. As Fig. 3c shows, the cross-shaped support region is not overlapped by the replicates, as long as the more dynamic portion of the FOV is restricted to one-quarter of the FOV only. This observation confirms the analysis of Kellman and McVeigh (9).

The graphical formalism reveals two solutions to overcome this quarter-FOV restriction. The first, described in this section, is a straightforward modification of SMASH.

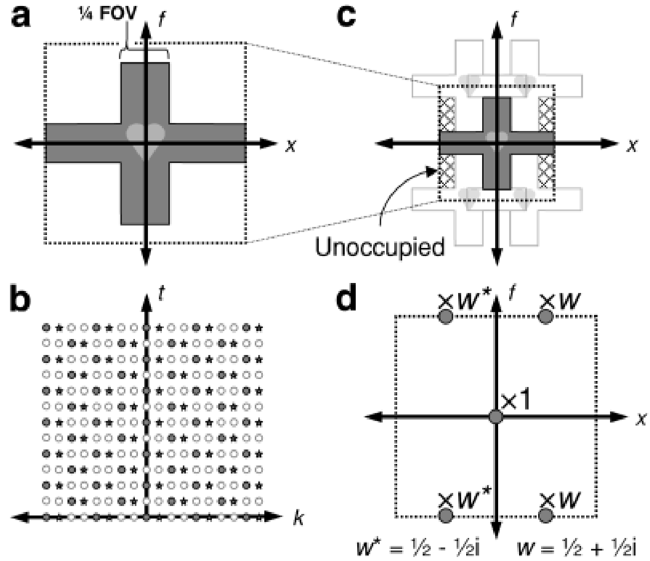

FIG. 3. UNFOLD combined with twofold SMASH acceleration. **a:** Shaded cross shape indicates the support region in *x-f*-space (*x*: spatial position along phase-encoding direction; *f*: temporal frequency). The signal intensity is assumed to be negligible outside the support region. The heart shape identifies the portion of *x* with a larger temporal bandwidth. **b:** The *k-t*-space sampling pattern with gray (●) and white dots (○) indicating sampled and skipped positions, respectively (*k*: *k*-space position along phase-encoding direction; *t*: time). The gray stars (★) indicate additional *k*-space data generated by SMASH. **c:** Periodic replication in *x-f*-space. Crosshatched areas are unoccupied. The size scale is reduced compared to **a**. **d:** The point spread function determining the replication pattern in **c**. Values in **d** indicate the relative weights of the point spread function.

The second involves a modification to UNFOLD, as described in the next section and demonstrated in the Examples section. In the first solution, for every sampled phase-encode line k_l, SMASH is used to generate the phase-encode line at both $k_l + 1$ and $k_l - 1$ (gray stars in Fig. 4b). The sampling pattern is shifted successively by one phase-encode line at each time frame (Fig. 4a). Using only the generated phase-encode lines (gray stars in Fig. 4b), the *k-t*-space sampling pattern is identical to that in Fig. 1b. Thus, applying the reconstruction procedure in Fig. 1 will recover all the remaining phase-encode lines (white and gray dots in Fig. 4b). However, only the recovered data at $k_l \pm 2$ are needed (white dots), while the sampled phase-encode lines k_l (gray dots) should be determined directly from the measured data to reduce error propagation.

In the second solution, the sampling pattern remains the same as that in Fig. 3b. The crosshatched areas in *x-f*-space (Fig. 3c) unoccupied by the replicates of the support region can be exploited to overcome the quarter-FOV restriction, as demonstrated in the following example. In Fig. 5a, the support region is assumed to be the same as that in Fig. 1a, with half of the FOV being more dynamic. To simplify subsequent description, the support region is segmented into five areas, labeled *a–e* (Fig. 5a). *k-t*-Space is sampled in the same pattern as in Fig. 3b, so the point spread function remains the same (Fig. 3d). In the zero-filled Fourier reconstruction (Fig. 5b), the replicates of the sup-

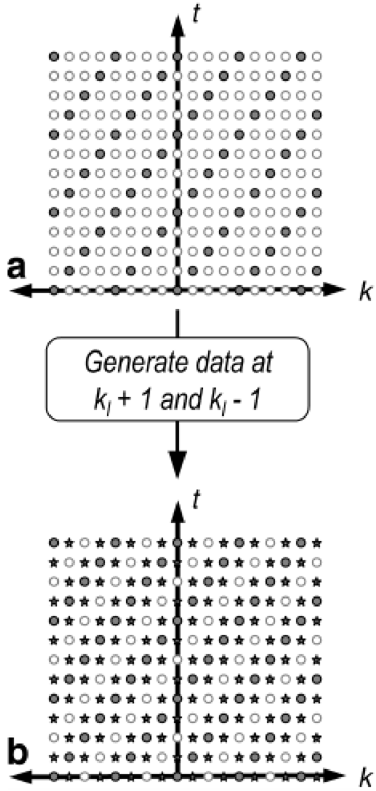

FIG. 4. UNFOLD combined with twofold SMASH acceleration. **a:** The *k*-*t*-space sampling pattern with gray (●) and white dots (○) indicating sampled and skipped positions, respectively (*k*: *k*-space position along the phase-encoding direction; *t*: time). k_l denotes the sampled phase-encode lines at each time point. **b:** SMASH is used to generate additional *k*-space data at $k_l \pm 1$, indicated by the gray stars (★). The generated phase-encode lines (★) alone form a sampling pattern identical to that in Fig. 1b.

port region overlap, so an unaliased copy of the signal cannot be recovered by simple filtering.

Although the areas *a*–*d* (shown in gray in Fig. 5b) are overlapped, there are corresponding areas in the replicates that are not overlapped (labeled A'–D'). Thus, an overlap-free copy of the signal can either be recovered from the unoverlapped areas (A'–D') or by solving the following linear system to unscramble the overlap:

$$\begin{bmatrix} 0 & 0 & w^* & 0 \\ 1 & 0 & 0 & w^* \\ 0 & 1 & w & 0 \\ 0 & 0 & 0 & w \\ w^* & 0 & 0 & 0 \\ 0 & w^* & 1 & 0 \\ w & 0 & 0 & 1 \\ 0 & w & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} I \\ II \\ III \\ IV \\ V \\ VI \\ VII \\ VIII \end{bmatrix} \qquad [1]$$

where *I*–*VIII* refer to the segmented areas of the zero-filled Fourier reconstruction (Fig. 5c), and *w* and w^* are equal to ½ + ½i and ½ − ½i, respectively, as determined by the relative weights in the point spread function (Fig. 3d). Slight noise amplification is expected in this reconstruction due to the inversion of the linear system (Eq. [1]). Nevertheless, the quarter-FOV restriction can be overcome. A similar analysis can be extended to higher SMASH acceleration factors, but the noise amplification will also increase.

Although the more dynamic portion of the FOV can occupy more than one-quarter of the FOV, it cannot occupy more than one-half of the FOV. This is because the total amount of available data (gray dots and stars in Fig.

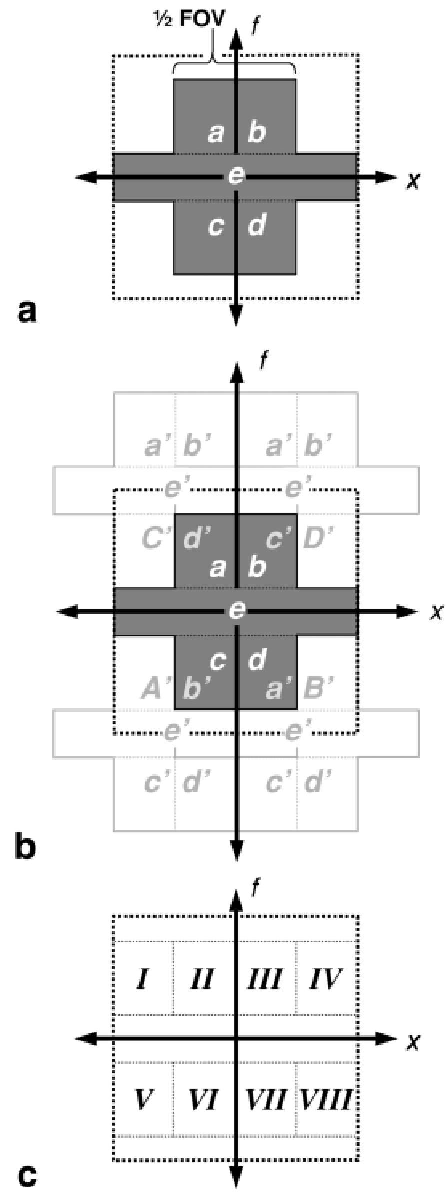

FIG. 5. UNFOLD combined with twofold SMASH acceleration. **a:** Shaded cross shape indicates the support region in *x*-*f*-space (*x*: spatial position along phase-encoding direction; *f*: temporal frequency). The signal intensity is assumed to be negligible outside the support region. The support region is divided into five areas labeled *a*–*e*. **b:** Periodic replication in *x*-*f*-space resulting from the *k*-*t*-space sampling pattern shown in Fig. 3b. Note that areas *a*–*d* are overlapped by replicates, which are divided into five areas labeled a'–e' in the same manner as in **a**. The unoverlapped areas of the replicates are labeled A'–D'. The *x*-*f*-space in **b** is segmented into various areas (*I*–*VIII*) according to **c** for subsequent algebraic manipulations to unscramble the signal overlap.

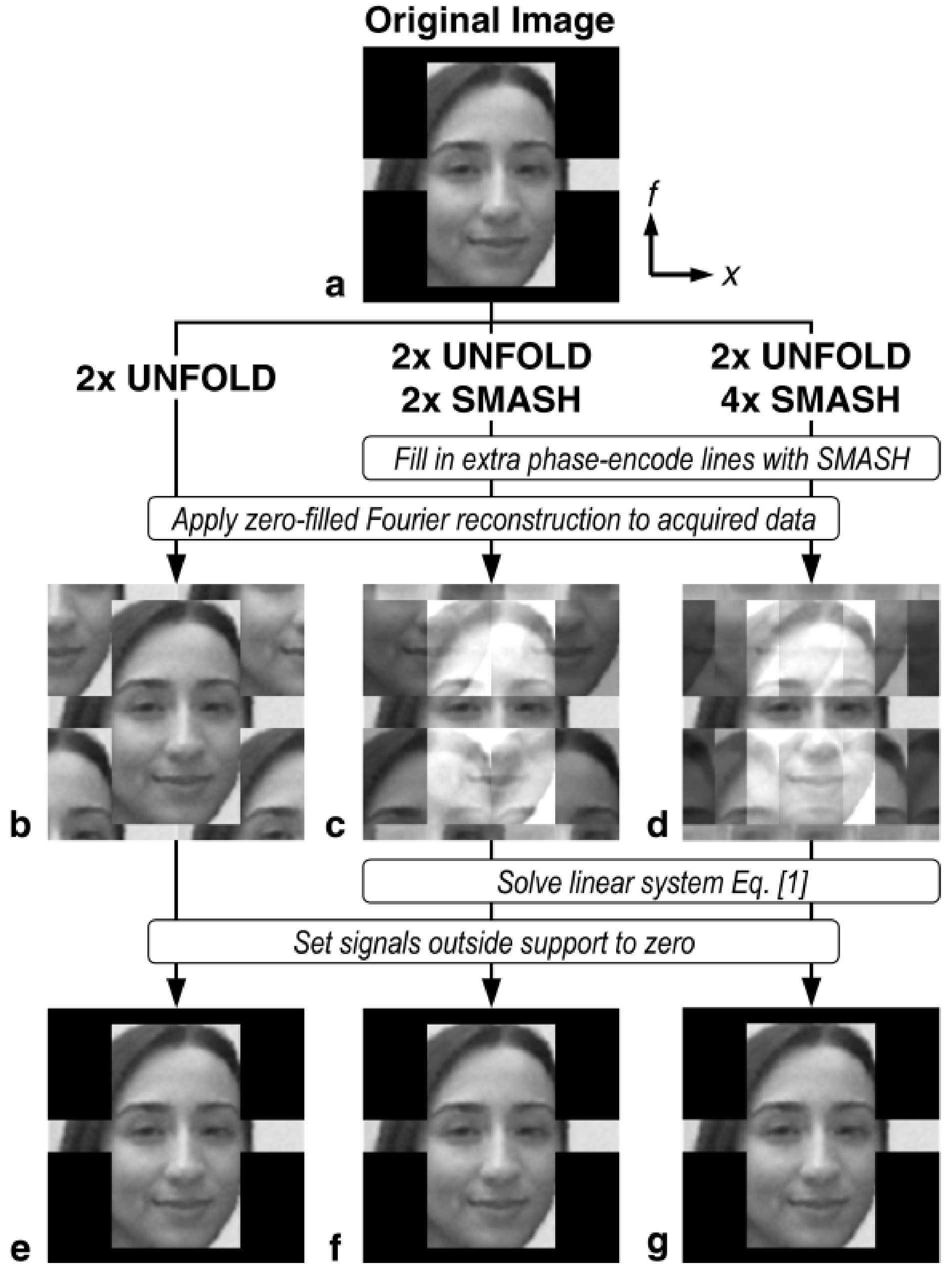

FIG. 6. UNFOLD and UNFOLD-SMASH. **a:** Original "data" in x-f-space with a cross-shaped support region (x: spatial position along phase-encoding direction; f: temporal frequency). The signal intensity is assumed to be negligible outside the support region. Data are collected in k-t-space according to the sampling pattern of UNFOLD alone (Fig. 1b), or combined with twofold (Fig. 3b) or fourfold SMASH acceleration. (k: k-space position along phase-encoding direction; t: time). **b–d:** Corresponding zero-filled Fourier reconstructions. **e:** For UNFOLD alone, signals outside the support region are set to zero to yield the final reconstruction. **f** and **g:** For UNFOLD combined with SMASH, additional algebraic manipulations are needed to yield the final reconstructions.

3b) takes up one half of k-t-space only, while the other half is skipped (white dots in Fig. 3b). Therefore, there are only enough data to reconstruct half of x-f-space. For a cross-shaped support region, this requirement is only satisfied if the more dynamic portion of the FOV occupies no more than half the FOV.

Examples

The following examples demonstrate the reconstruction algorithms presented in the Theory section for the cardiac application of UNFOLD. A photograph is used to represent the data in x-f-space because the image features of the photograph make it easier to recognize the aliasing pattern. Figure 6a shows the true "data" in x-f-space, with a cross-shaped support region. Two-, four-, and eightfold acceleration factors were achieved by applying UNFOLD alone, or in combination with two- or fourfold SMASH acceleration (to generate adjacent phase-encode lines). The corresponding zero-filled Fourier reconstructions are shown in Fig. 6b–d, respectively. For the reconstructed image from UNFOLD alone, an unaliased copy of the signal can be recovered by setting all signals outside the support region to zero (Fig. 6e). For those from UNFOLD combined with SMASH, additional algebraic manipulation is needed to recover the unaliased signals (Fig. 6f and g), as outlined in Eq. [1].

CONCLUSIONS

In summary, UNFOLD is an elegant method for speeding up data acquisition without requiring special hardware. A graphical formalism is presented, which illustrates the principles of UNFOLD by analyzing its sampling pattern in *k*-*t*-space. This formalism shows that the maximum acceleration achievable by UNFOLD is equivalent to the maximum packing of support regions in *x*-*f*-space. When the support region is separable along the *x* and *f* axes, the reconstruction can be performed separately for each *k*.

The graphical formalism also provides new insights into the combined method of UNFOLD and SMASH. Two solutions are presented which allow the more dynamic portion of the FOV to occupy up to half the FOV.

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