MAGNETIC RESONANCE IMAGING

Physics

"Nuclear Magnetic Resonance"
discovered independently by **Bloch & Purcell**

"Classical Description" (full desc. requires QM)

atoms w/ odd # protons and/or neutrons

Spin angular momentum $\vec{S} = \hbar \vec{I}$

$\hbar$: $\frac{\text{Planck's const}}{2\pi}$

$\vec{I}$: spin quantum number

magnetic dipole moment $\vec{\mu} = \gamma \vec{S}$

γ: gyromagnetic ratio

charged spinning sphere $= \gamma \hbar \vec{I}$

Atoms? ^{1}H, ^{31}P, ^{23}Na, ^{13}C

most abundant

"Spins"

Magnetic Fields

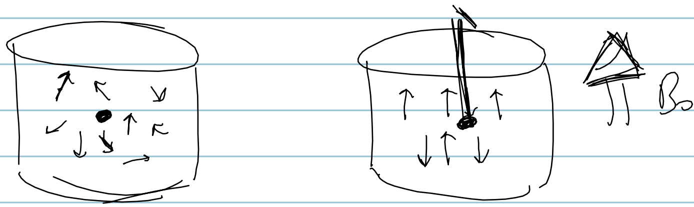
$\vec{\mu}$

B_0 — static, v. strong

B_1 — radio frequency

$G_{x,y,z}$ — spatial encoding

$\boxed{B_0}$ Polarization : achieves macroscopic magnetization



energy $\vec{\mu}$ in $\vec{B}_0$ $= -\vec{\mu}\cdot\vec{B} = -\mu_z B_0$

$= -\gamma\hbar I B_0$

$= \pm \frac{\gamma\hbar B_0}{2}$

$^1H = \pm 1/2$

two energy states

ΔE

n^- antiparallel (higher energy)

n^+ parallel

$$\gamma\hbar B_0 = h\underbrace{\frac{\gamma}{2\pi}B_0}_{f_0 \,=\, \text{Larmor frequency}}$$

Boltzmann distribution

$$\frac{n^-}{n^+} = e^{-\frac{\Delta E}{kT}}$$

energy diff

absolute temperature

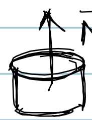
Boltzmann const

@ ^1H, Room Temp, 1.5 T

$$\frac{n^-}{n^+} \cong 0.99993$$

"weak" polarization

basis for almost all MRI.



$$\vec{M} = \sum \vec{\mu}$$

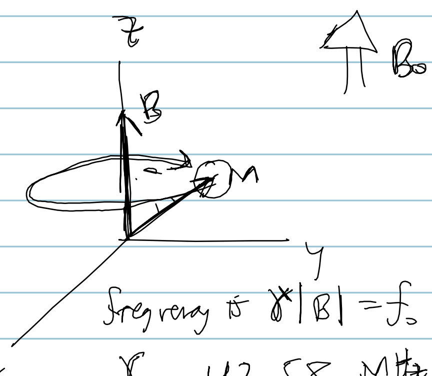
Establishes Resonance Condition

@ equilibrium $\vec{M} \parallel \vec{B_0}$

If $\vec{M} \not\parallel \vec{B_0}$ there is an induced torque

$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma \vec{B}$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$



$\odot$ out of the board

$\vec{M} \times \vec{B}$

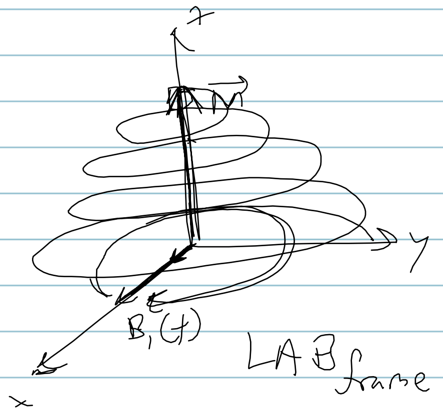
θ is preserved

frequency is $\gamma |B| = f_0$

$$\frac{\gamma}{2\pi} = 42.58 \text{ MHz/T}$$

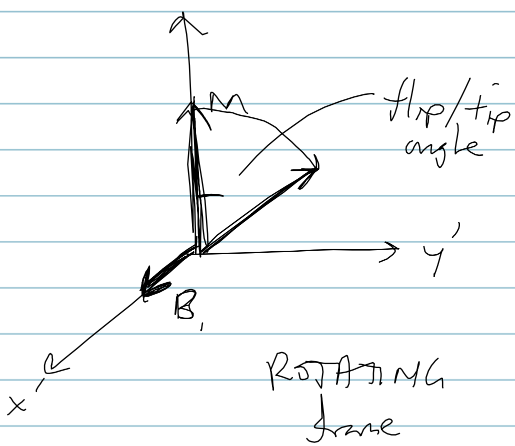
B_0 provided polarization & resonance conditions

$\boxed{B_1}$ radiofrequency γB_0 to excite nuclei
to listen



apply rotating $B_1(t)$ in the transverse plane at f_0

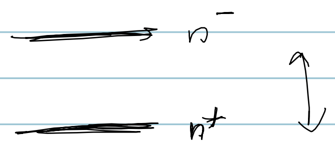
perturb nuclei



x', y' rotating @ f_0

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma(\vec{B}_{eff})$$

another look:

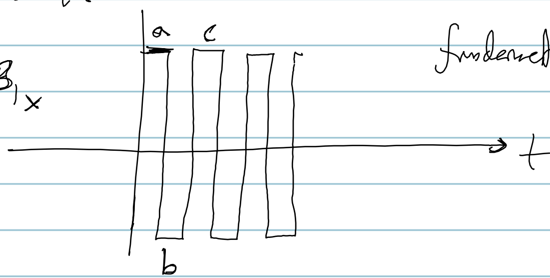


$$\Delta E = h \frac{\gamma}{2\pi} B_0 = h f_0$$

applying RF @ f_0 → introduce energy exchange between these two states

yet another look:

consider $B_{1,x}$



fundamental freq, f_0

LAB frame