Another derivation of the Signal Equation

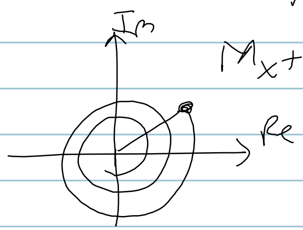
$$S_r(t) = \int m(x,y,t)\,dx\,dy$$

slice (plane)

ignore T_2

$$= \int_{x,y} m(x,y)\, e^{-i\phi(x,y,t)}\,dx\,dy$$

$M_x + iM_y$



$\phi(x,y,t)$??

$$\omega(t) = \frac{d\phi(t)}{dt}$$

$$\therefore\ \phi(t) = \int_0^t \omega(\tau)\,d\tau$$

$$\omega(x,y,t) = \gamma\, \underbrace{B_z(x,y,t)}_{\text{includes } B_0 \,\&\, G}$$

$$= \gamma\left(B_0 + \underbrace{G_x(t)\,x + G_y(t)\,y}_{\text{control}}\right)$$

$$\phi(x,y,t) = \gamma \int_0^t B_0 + G_x(\tau)x + G_y(\tau)y \, d\tau$$

$$= \omega_0 t + 2\pi \underbrace{\left(\frac{\gamma}{2\pi}\int_0^t G_x(\tau)\,d\tau\right)}_{k_x(t)} x + 2\pi \underbrace{\left(\frac{\gamma}{2\pi}\int_0^t G_y(\tau)\,d\tau\right)}_{k_y(t)} y$$

$$= \omega_0 t + 2\pi\left(k_x(t)x + k_y(t)y\right)$$

$$S_r(t) = \iint_{x\,y} m(x,y)\, \cancel{e^{-i\omega_0 t}}\, e^{-i2\pi(k_x(t)x + k_y(t)y)}\, dx\,dy$$

baseband

$$s(t) = s_r(t)\, e^{+i\omega_0 t} = M\left(k_x(t), k_y(t)\right)$$

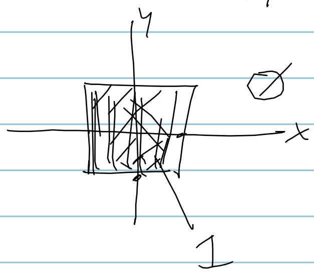
key points

1) using G_x & G_y to control the instantaneous frequency at different x,y positions $\longrightarrow$ imparts a space dependent phase

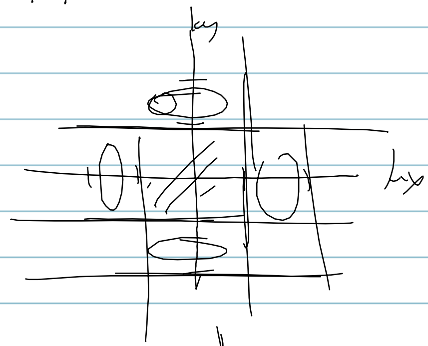
2) <u>linear gradients</u> $\longrightarrow$ impart a linear phase variations

3) phase $\phi = \underbrace{2\pi \overbrace{k_x(t)}^{\text{cycles per unit distance}}}_{\text{phase per unit distance}} x + \cdots$

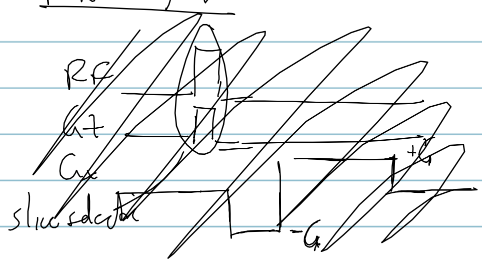
consider $m(x,y) = {}^{2}\Pi(x,y)$

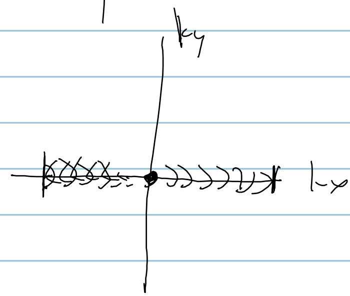


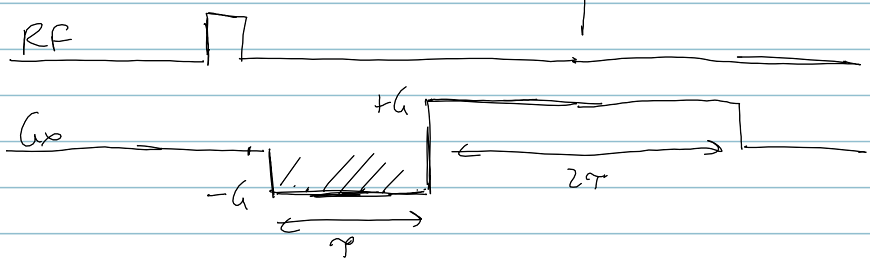
$M(k_x, k_y) = \text{sinc}(k_x)\,\text{sinc}(k_y)$

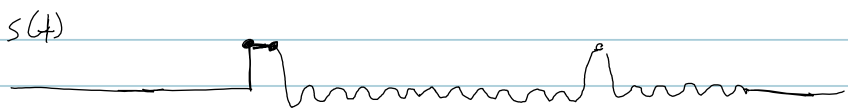


Pulse sequence









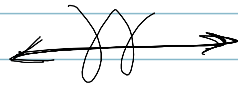
$\left(-\frac{\gamma}{2\pi} G\tau \, , \, \frac{\gamma}{2\pi} G\tau \right)$ range covered along k_x

$\boxed{\text{BREAK}}$

complex demodulation

$$s(t) = s_r(t)\, e^{+i\omega_0 t}$$

$$s_r(t) = \underline{\alpha(t)}\, e^{-i(\omega_0 t + \underline{\phi(t)})}$$



physical signal

$$s_p(t) = \mathrm{Re}\{ s_r(t) \}$$

$$= \underline{\alpha(t)} \cos(\omega_0 t + \underline{\phi(t)})$$

$$= \underline{\alpha(t)\cos\phi(t)} \cos\omega_0 t - \underline{\alpha(t)\sin\phi(t)} \sin\omega_0(t)$$

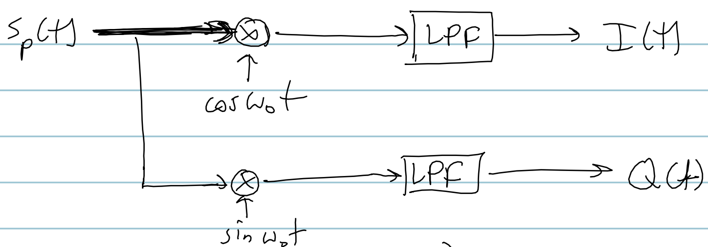
orthogonal & in quadrature

<u>want</u> baseband signal

$$s(t) = \alpha(t)\, e^{-i\phi(t)}$$

$$= \underbrace{\alpha(t)\cos\phi(t)}_{I(t)} - i\underbrace{\alpha(t)\sin\phi(t)}_{Q(t)}$$

<u>Quadrature Phase Sensitive Detection (QPSD)</u>



this is how it is done !!

$$\left(\alpha(t)\cos(\omega_0 t + \phi(t))\cos\omega_0 t\right) * \underset{\text{LPF}}{h(t)} \Longrightarrow \alpha(t)\cos\phi(t)$$