Excitation Chapter 6

1) non-selective (G off)

2) selective excitation (G on)

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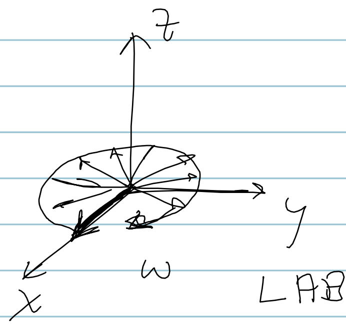
non selective excitation



RF field B @ carrier freq ω

envelope $B_1(t)$

desire $\omega = \omega_0$

$$B_1(t)\left(\cos\omega t\,\hat{i} - \sin\omega t\,\hat{j}\right)$$

Bloch Eq'n : $\frac{d\vec{m}}{dt} = \vec{m} \times \gamma \vec{B}$

$$\vec{B} = B_0\hat{k} + B_1(t)\left(\cos\omega t\,\hat{i} - \sin\omega t\,\hat{j}\right)$$

Rotating Frame   rotates around z axis
with frequency $\underline{\omega}$

$$\vec{M}_{rot} = \begin{bmatrix} M_x' \\ M_y' \\ M_z \end{bmatrix} \qquad \vec{B}_{rot} = \begin{bmatrix} B_x' \\ B_y' \\ B_z \end{bmatrix}$$

$$\vec{M}_{lab} = R_z(\omega t)\,\vec{M}_{rot}$$

$$\vec{B}_{lab} = R_z(\omega t)\,\vec{B}_{rot}$$

Simplify Bloch in Rot Frame (derivation in Ch 6 App I)

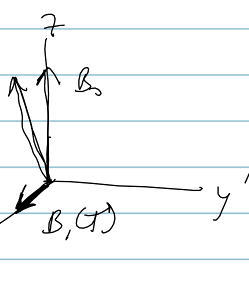
$$\frac{d\vec{m}_{rot}}{dt} = \vec{m}_{rot} \times \left[\gamma \vec{B}_{rot} - \omega \hat{k}\right] \quad *$$

$$\vec{B}_{eff} \triangleq \vec{B}_{rot} - \frac{\omega}{\gamma}\hat{k}$$

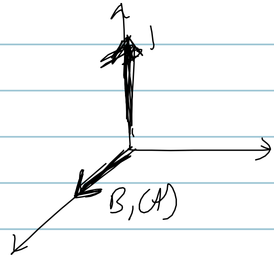
$$\frac{d\vec{m}_{rot}}{dt} = \vec{m}_{rot} \times \gamma \vec{B}_{eff}$$

$$\vec{B}_{rot} = B_0 \hat{k} + B_1(t)\,\hat{i}$$

last page



$\vec{B}_{eff}$



$\omega = \omega_0$

when $\omega = \omega_0$

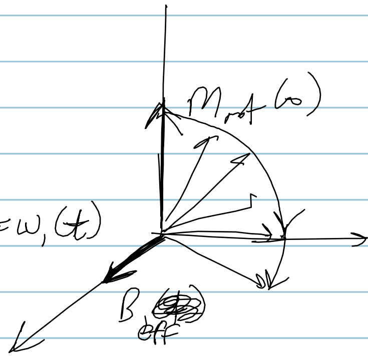
$$= \left(\cancel{B_0 \hat{k}} + B_1(t)\,\hat{i}\right) - \cancel{\frac{\omega}{\gamma}\hat{k}}$$

$$= B_1(t)\,\hat{i}$$

freq of rotation

$$\gamma\,|B_{eff}| = \gamma B_1(t) = \omega_1(t)$$

$$\theta(t) = \int_0^t \omega_1(\tau)\,d\tau$$

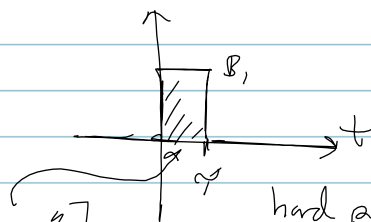


"flip angle"

"tip angle"

$$B_1(t) = \begin{cases} B_1 & 0 < t < \tau \\ 0 & \text{else} \end{cases}$$

$$\omega_1 = \gamma B_1$$



hard pulse

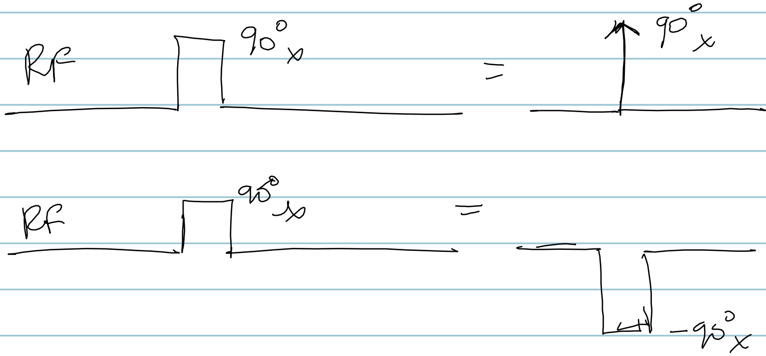
$$\theta = \gamma B_1 \tau$$

$$= \gamma\,[\text{"area under } B_1(t)\text{"}]$$

Terms: 1) tip angle θ

2) axis of rotation $\leftarrow$ B_{eff} can be oriented along any transverse axis

Pulse Sequence diagrams



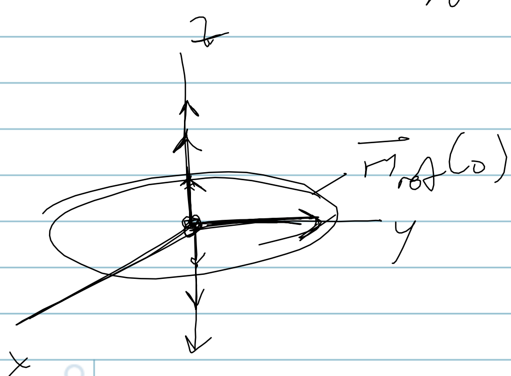
Example #2: gradient, no RF

$\vec{B} = (B_0 + G_x x)\hat{k}$

rot frame — ω_0

$\vec{B}_{rot} = \vec{B}$

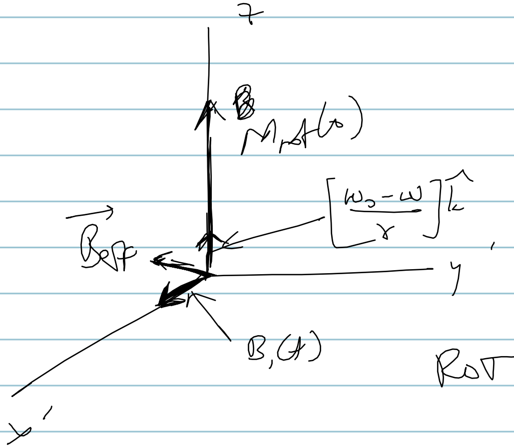
$\vec{B}_{eff} = [\cancel{B_0} + G_x x]\hat{k} - \cancel{\frac{\omega}{\gamma}}\hat{k} = G_x x\hat{k}$



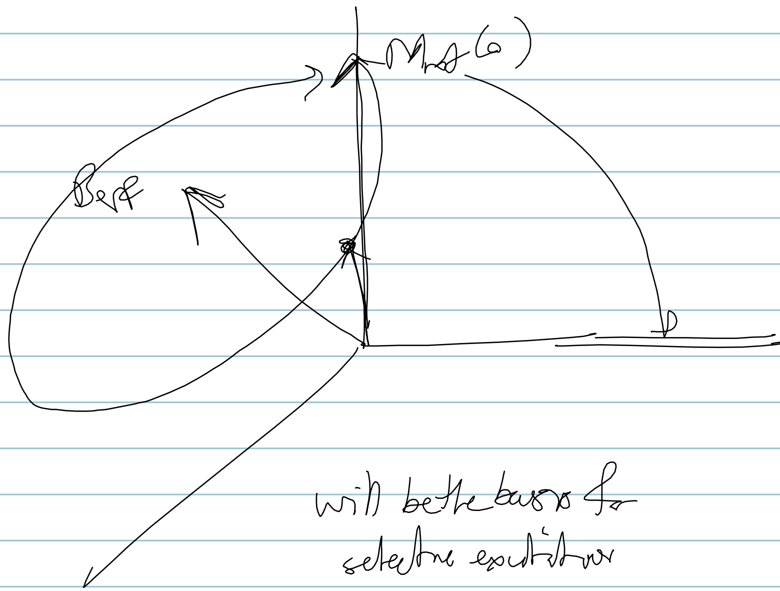
G off
B_1 on

what if $\omega \neq \omega_0$

$$B_{eff} = \left[\frac{\omega_0 - \omega}{\gamma}\right]\hat{k} + \underline{B_1(t)\,\hat{i}}$$



Rot

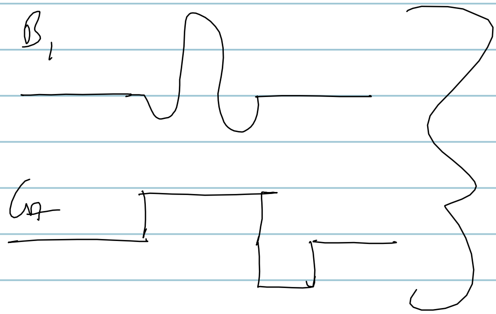


will be the basis for
selective excitation

$\vec{G}$ creates ~~$\omega_0 - \omega$ mis~~ $\Delta\omega$ mismatch at
all positions except for a slice of interest

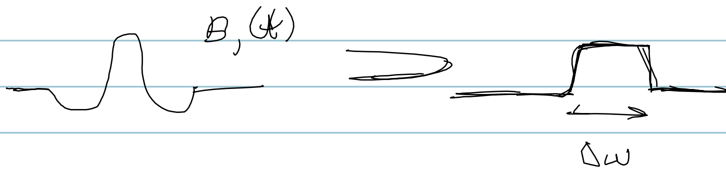
B_1 + G_z simultaneously

$\boxed{\text{selective excitation preview}}$

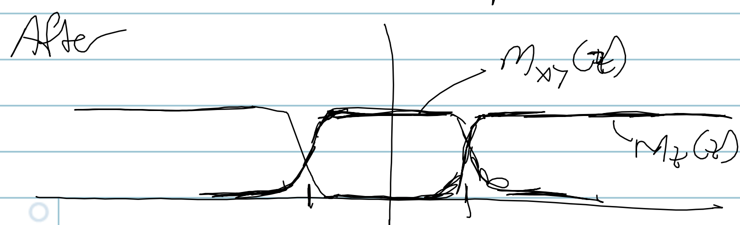


excites a slice $\perp$ to z axis

slice profile $M_{xy}(z)$ depends on the $\mathcal{F}\{B_1(t)\}$



slice profile plots $M_{xy}(z)$, $M_z(z)$



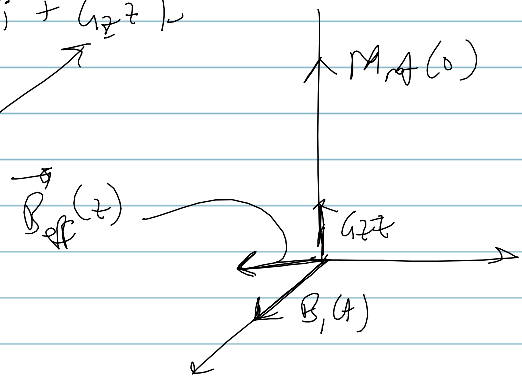
$B_1(t)$ & G_z

$$\vec{B}_{eff} = \begin{bmatrix} B_1(t) \\ \emptyset \\ \cancel{\frac{\omega_0-\omega}{\gamma}} + G_z z \end{bmatrix} \qquad \text{let } \omega = \omega_0$$

$$\vec{B}_{eff} = B_1(t)\,\hat{i} + G_z z\,\hat{k}$$

$\omega_1(t) = \gamma B_1(t)$

$\omega(z) = \gamma G_z z$



1) $z = \emptyset$ — $\vec{M}_{rot}$ rotates within the y-z plane

2) z large — $\vec{B}_{eff}$ almost // to z-axis
$|G_z z| \gg |B_1(t)|$ — no tip. no rotation

3) z intermediate — some smaller/intermediate tipping

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

$$\frac{d\vec{M}_{rot}}{dt} = \begin{bmatrix} 0 & \gamma G_z z & 0 \\ -\gamma G_z z & 0 & \gamma B_1(t) \\ 0 & -\gamma B_1(t) & 0 \end{bmatrix} \vec{M}_{rot}$$

$\omega(z)$ $\omega_1(t)$

solve for transverse magnetization

$$M_r = M_x' + i M_y'$$

$$\frac{dM_r}{dt} = \frac{dM_x}{dt} + i \frac{dM_y}{dt}$$

$$= \left(\gamma G_z z M_y'\right) + i\left(-\gamma G_z z M_x' + \gamma B_1(t) M_z'\right)$$

$$= \gamma G_z z \left(-i M_r'\right) + i \gamma B_1(t) M_z'$$

$$\frac{dM_r}{dt} = -i\,\omega(z) M_r' + i\,\omega_1(t) M_z'$$

$$\frac{dM_z}{dt} = -\omega_1(t) M_y'$$

two coupled DE's

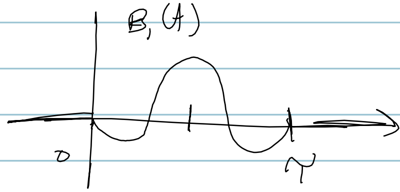
Small Tip Approximation

$\theta \le 30^\circ$, $\underline{M_z \approx M_0}$, $\frac{dM_z}{dt} \approx 0$

$$\frac{dM_r}{dt} = -i\,\omega(z) M_r + \omega_1(t) M_0$$

full solution Ch6, Appx 2

$$M_r(t,z) = i M_0 e^{-i\omega(z)t} \int_0^t \omega_1(s) e^{+i\omega(z)s}\, ds$$

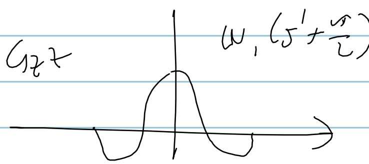


let $s' = s - \frac{\tau}{2}$

$$M_r(\tau, z) = i M_0 e^{-i\omega(z)\tau} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \underline{\omega_1(s' + \tfrac{\tau}{2})}\, e^{i\omega(z)s'} e^{i\omega(z)\frac{\tau}{2}}\, ds'$$

$$= i M_0 e^{-i\omega(z)\frac{\tau}{2}}$$

$$\mathcal{F}\{\omega_1(t + \tfrac{\tau}{2})\}\Big|_{f = -\frac{\gamma}{2\pi} G_z z}$$



Check it!

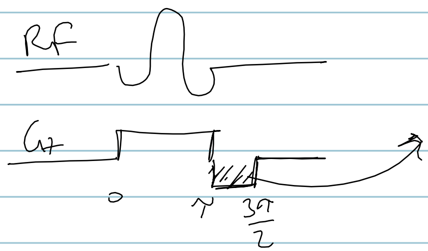
$z = 0$

$$M_r(\tau, 0) = i M_0 \int \omega_1(s)\,ds = i M_0 \theta$$

expect $i M_0 \sin\theta$

small tip approx $\sin\theta \approx \theta$

Refocusing b/c $e^{-i\omega(z)\tau/2}$



phase shift

$$-\gamma \int_{\tau}^{\frac{3\tau}{2}} -G_z z \, dt'$$

$$= \gamma G_z z \frac{\tau}{2}$$

$$= \omega(z) \frac{\tau}{2}$$

$$M_r\left(\frac{3\tau}{2}, z\right) = M_r(\tau, z)\, e^{+i\omega(z)\frac{\tau}{2}}$$

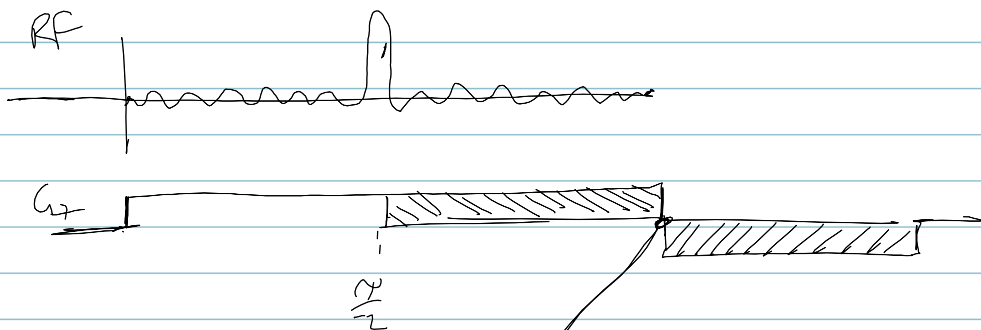
$$= i M_0 \mathcal{F}\left\{\omega_1\left(t + \frac{\tau}{2}\right)\right\}$$

$$M_r\left(\frac{3\tau}{2}, z\right) = , M_0 \mathcal{F}\left\{\omega_1\left(t+\frac{\tau}{2}\right)\right\}$$

if $\omega_1\left(t+\frac{\tau}{2}\right)$ "even"

all excited spins are in-phase

extreme example of refocusing



2D imaging

$$m(x,y) = \int_{-\frac{\Delta z}{2}}^{\frac{\Delta z}{2}} m(x,y,z) \cancel{e^{-i\phi z}} \, dz$$

dephased signal