

EE591
Project Report
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Two Dimensional Spatially Selective RF Pulse Design Using Spiral Trajectory

Abstract

In this report we discuss the design of a two dimensional spatially selective RF pulse for a pencil beam excitation, given a resolution and Field of View(FOV). We take into consideration the hardware limitations for gradient and gradient slew rate. We begin with the design of the requisite spiral k-space trajectory and the corresponding gradients followed by the design of the weighting function and the RF pulse. We also study the effect of different k space weighting functions, gradient delays and off resonance on the excitation profile. The report concludes with the design and evaluation of a variable density spiral k-space trajectory.

Introduction

The application of an RF field (B_1) in the presence of B_0 alone excites all the spins in the volume that are sensitive to the excitation coil. The excitation of a portion of the volume can be achieved by the application of an RF pulse with modulation function $B_1(t)$ in the presence of gradient fields (G_x and/or G_y and/or and/or G_z). Those spins having resonant frequencies within the bandwidth of $B_1(t)$ are excited, which are imaged later on using suitable phase encoding and frequency encoding gradients [1]. We can compute the excitation profile by solving the coupled differential equations arising from the Bloch equation. But such coupled differential equations are difficult to solve explicitly. Hence to decouple to such equations we assume that the initial magnetization is $[0 \ 0 \ M_0]$ and that the RF pulse is “weak” leading to a small tip angle. The solution of the resulting simplified equations gives the excitation profile. All the derivations in this project are based on small tip excitation.

Multidimensional spatially selective RF pulses are used for restricting the area under consideration by reducing the field of view. By using different k-space trajectories we can achieve specific excitation patterns like cylinders and cubes. Two-dimensional spatially selective radio-frequency pulses that are based on spiral gradient waveforms are used to excite a pencil-beam-shaped region. The desired resolution and FOV can be achieved by appropriately selecting the spiral properties like maximum radius, number of turns, density of the spiral etc. This project deals with spatially selective RF pulses using spiral trajectories. All the design steps and equations are taken from [2].

Objectives:

1. To design an RF waveform that excites a cylinder of diameter 4 cm (with resolution = 1cm and FOV = 16 cm),
 - Using spiral k-space trajectory sampled at non-uniform angular rate to use the gradient system more efficiently,
 - with constant k-space weighting function.
 - with windowed jinc k-space weighting function.
2. To plot the excitation profiles with the designed RF pulses.
3. To shift the selected excited area by 2 cms in x direction and 2 cms in the y direction.
4. To study the effect of gradient delay on the excitation profile.
5. To study the effect of off-resonance on the excitation profile.
6. To design a variable density spiral and to study its effects on the excitation profile.

Theory:

The solution of Bloch Equation using small tip angle approximation is given by,

$$M_{xy}(r, t) = iM_0 \int_{-\infty}^t \gamma B_1(\tau) \exp(i2\pi k(\tau, t) \cdot r) d\tau \quad \text{----- (1)}$$

This can be written as,

$$\begin{aligned} M_{xy}(r, t) &= iM_0 \int_{-\infty}^t \gamma B_1(\tau) \int_k \delta(k(\tau, t) - k) \exp(i2\pi k \cdot r) dk d\tau \\ &= iM_0 \int_k \left[\int_{-\infty}^t \gamma B_1(\tau) \delta(k(\tau, t) - k) d\tau \right] \exp(i2\pi k \cdot r) dk \\ &= iM_0 \int_k P(k) \exp(i2\pi k \cdot r) dk \quad \text{----- (2)} \end{aligned}$$

From equation (2) we get that the magnetization is the inverse transform of P(k). where,

$$P(k) = W(k)S(k)$$

where,

$$W(k(\tau, t)) = \frac{\gamma B_1(\tau)}{|k'(\tau, t)|} \quad \text{----- (3)}$$

$$S(k) = \int_{-\infty}^t \delta(k(\tau, t) - k) / k'(\tau, t) d\tau \quad \text{----- (4)}$$

$W(k(\tau, t))$ is the k-space weighting and $S(k)$ is the k space sampling function.

where “ t ” varies from 0 to 1 in constant steps of “ $1/1023$ ”.i.e there are 1024 sample points. The impulse response of the spiral can be computed by taking the Fourier inverse of the trajectory (fig2).

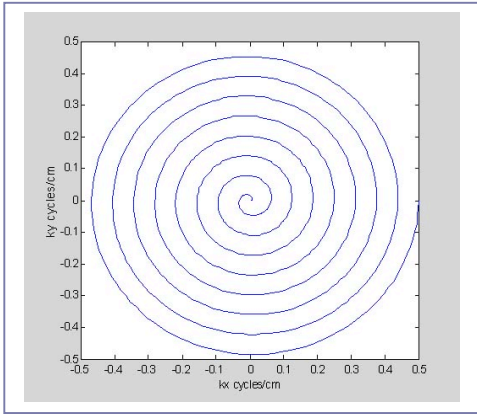


Fig 1. Spiral Trajectory, N=8turns, FOV=16cm, $\Delta r=1$ cm

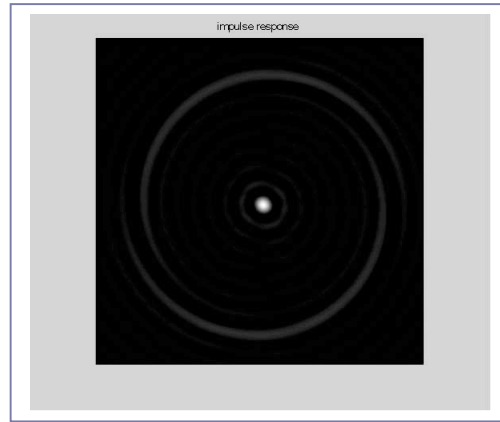


Fig 2. Impulse response of the trajectory given in fig1

“ k_x ” is the real part of “ kr ” and “ k_y ” is the imaginary part of “ kr ”. “ k_x ” and “ k_y ” are shown in fig 3 and fig4 respectively.

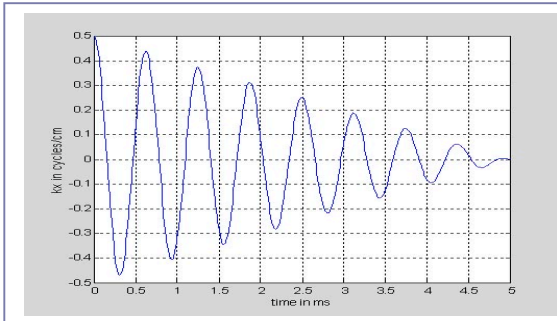


Fig 3. k_x in cycles /cm of the trajectory designed in fig 1.

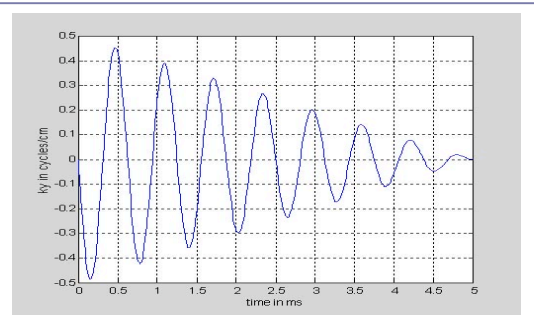


Fig 4. k_y in cycles/cm of the trajectory designed in fig 1.

Design of 2D Gradients:

$k_x(t)$ and $k_y(t)$ are defined in terms of gradients G_x and G_y respectively as given below,

$$k_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(s) ds \quad \text{and} \quad k_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(s) ds$$

Therefore the gradients can be calculated from k_x and k_y as,

$$G_x(t) = \frac{2\pi}{\gamma} k_x'(t) \quad \text{and} \quad G_y(t) = \frac{2\pi}{\gamma} k_y'(t)$$

Slew rate (S_x and S_y) of the gradient pulses are the derivative with respect to time of the gradients and hence they can be calculated from k_x and k_y .

$$S_x(t) = \frac{2\pi}{\gamma} k_x''(t) \quad \text{and} \quad S_y(t) = \frac{2\pi}{\gamma} k_y''(t)$$

The gradient waveforms and the slew rates for the designed spiral trajectory are given in fig (5) and fig (6) respectively.

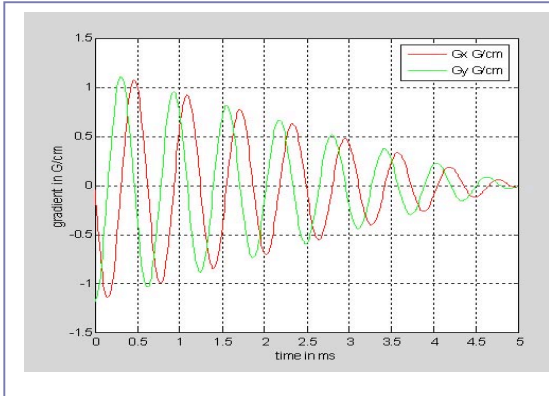


Fig 5. Gradient waveforms, G_x G/cm and G_y G/cm for the constant angular rate spiral of fig 1.

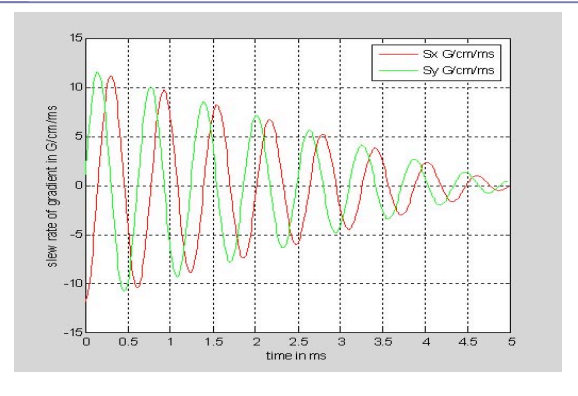


Fig 6. Gradient slew rate waveforms, s_y G/cm/ms and s_x G/cm/ms for the constant angular rate spiral

From fig (5) and fig (6) we can see that the maximum values of the gradients and the slew rates are reached only at one end. But for greater efficiency we would want the gradients or the slew rates to always be at the maximum values during the entire pulse. Thus, this design does not efficiently use the hardware capabilities. If the rate at which the spiral trajectory is traversed is changed the gradient waveforms will also change. Hence, efficiency can be improved by using non uniform angular sampling rate so as to speed up the trajectory when it is below the constraint and slow it down when it is over it. For designing a non uniform angular rate spiral a matlab mfile from [2] has been used. The designed spiral, k_x and k_y , gradient waveforms and the slew rates are given in fig (7),fig (8), fig(9) and fig(10) respectively. The gradients designed here are not very accurate because practically they should begin and end at zero. These gradients can be made to end at zero by ramping it over to zero.

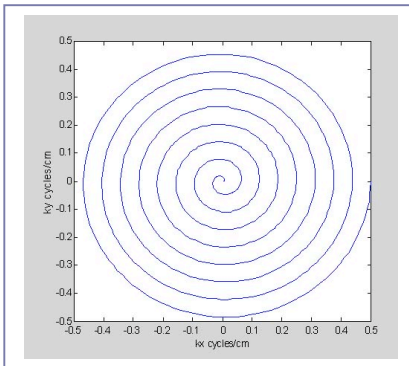


Fig 7. Spiral Trajectory with variable angular rate sampling

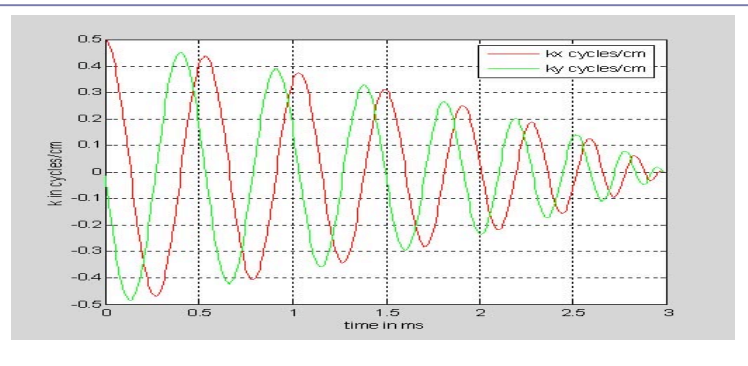


Fig 8. k_x (cycles/cm) and k_y (cycles/cm) waveforms for the trajectory designed in fig 7

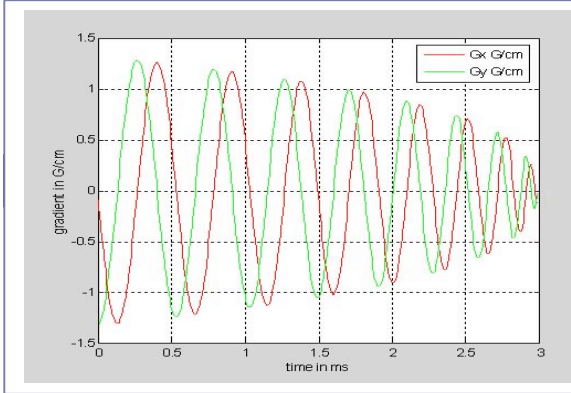


Fig 9. Gradient waveforms, G_x (G/cm) and G_y (G/cm) for the trajectory given in fig 7.

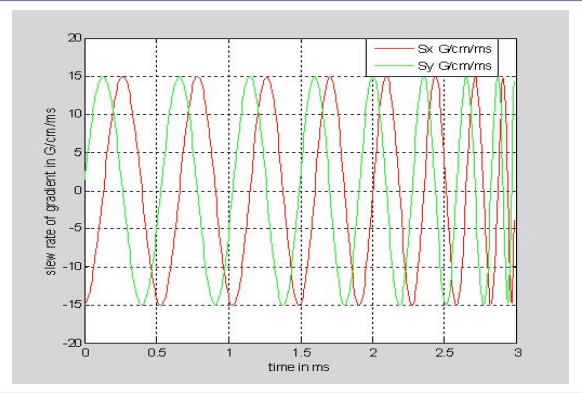


Fig 10. Gradient slew rate waveforms, s_x (G/cm/ms) and s_y (G/cm/ms) for trajectory in fig 7

Design of weighting Function:

The weighting function deposits RF energy at discrete locations in the k-space. In this project we have designed RF pulse for two different weighting functions.

1. uniform k space weighting:

$W(kr) = \text{rect}\left(\frac{kr}{2k_{\max}}\right)$, i.e. the weighting is one for all the points in k space (fig11). This type of weighting generates excessive ringing.

2. windowed jinc k space weighting:

The windowed jinc weighting used here are of space bandwidth 4 and 8. The weighting function with space bandwidth of 4 (fig 12) is given by,

$$W(kr) = \text{jinc}\left(\left| \frac{kr}{k_{\max}} \right| \right) (0.54 + 0.46 \cos(\pi \left| \frac{kr}{k_{\max}} \right|))$$

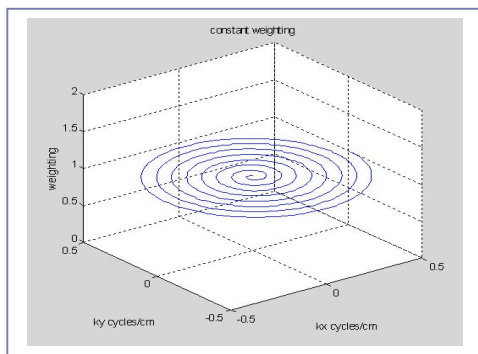


Fig 11. Uniform k space weighting function

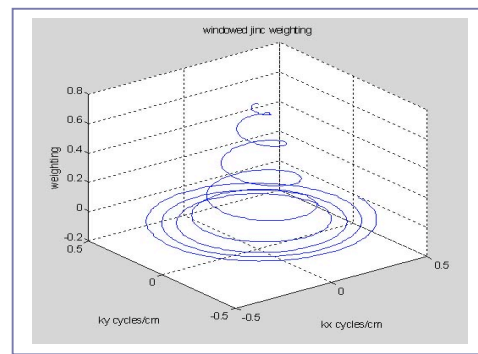


Fig 12. Windowed jinc k space weighting function

Design of RF Pulse:

1. For uniform k space weighting

From equation (5) we have

$$B1(\tau) = |G(\tau)| / W(k(\tau, t)) = |G(\tau)|$$

i.e. in the case of uniform k space weighting the RF pulse is simply the absolute value of the gradients used. The designed RF pulse is given in fig 13.

2. For windowed jinc k space weighting

From equation (5) we have

$$B1(\tau) = |G(\tau)| / W(k(\tau, t))$$

Here,

$$W(kr) = jinc\left(\left|kr\right| \frac{2}{k \max}\right) \left(0.54 + 0.46 \cos\left(\pi \frac{|kr|}{k \max}\right)\right)$$

Therefore,

$$B1(\tau) = |G(\tau)| jinc\left(\left|kr\right| \frac{2}{k \max}\right) \left(0.54 + 0.46 \cos\left(\pi \frac{|kr|}{k \max}\right)\right)$$

The designed RF pulse is given in fig 14.

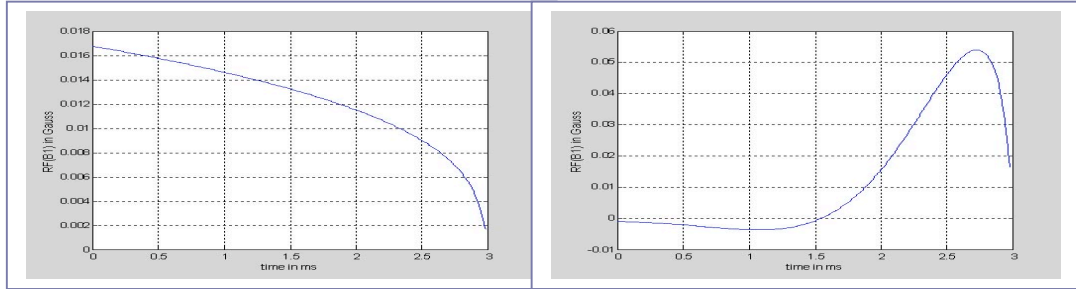
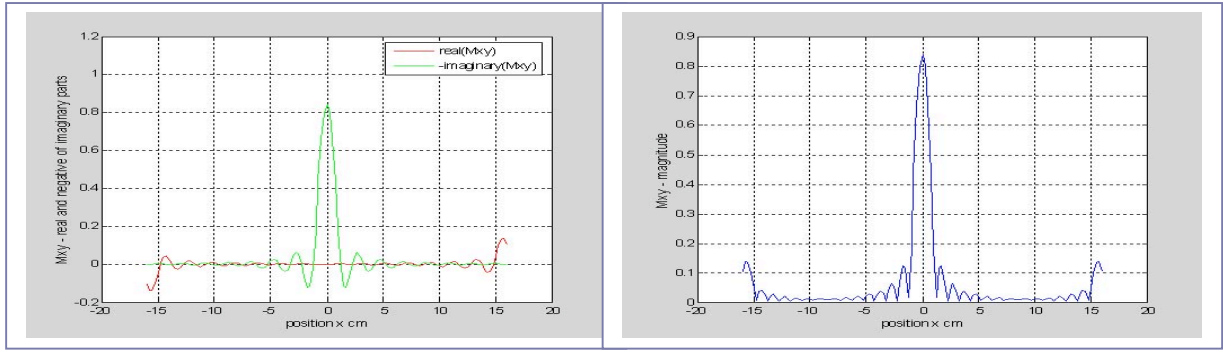


Fig 13. Designed RF pulse (with uniform weighting)

Fig 14. Designed RF pulse (with windowed jinc weighting)

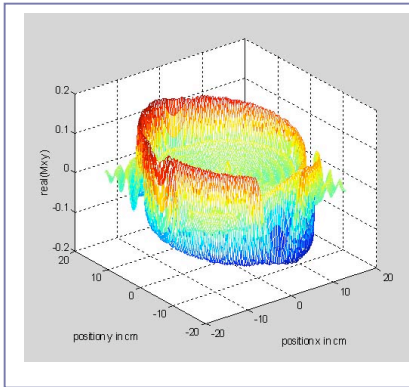
Computation of Excitation Profile:

The excitation profiles were generated using a matlab function given on [2]. This function takes the normalized designed RF and the gradients as input and using spinors calculates the excitation profile. The excitation profile for the RF with uniform k space weighting is given in fig 15 and that for the RF with windowed jinc weighting is given in fig 16. The RF pulse was designed for exciting a cylinder of 4 cm diameter.

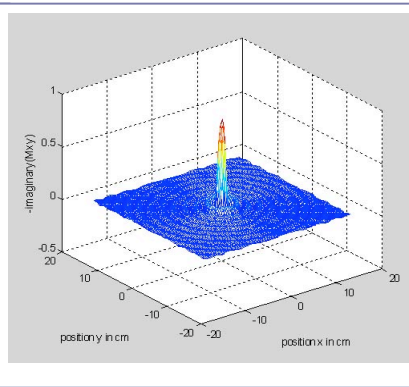


(a)

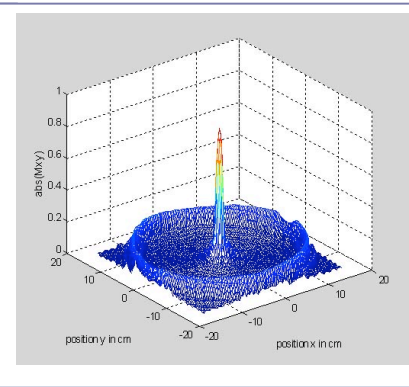
(b)



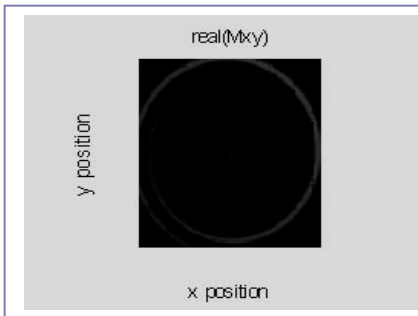
(c)



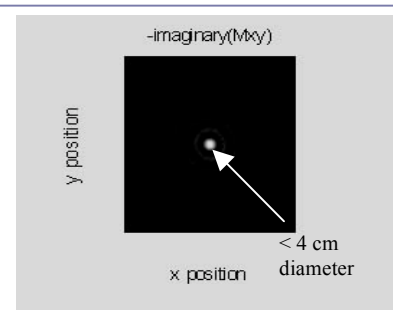
(d)



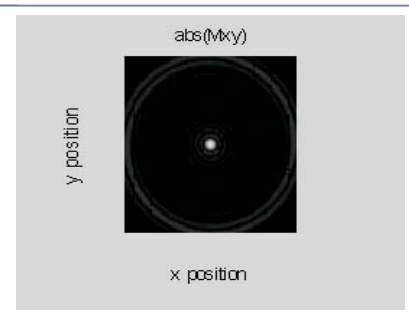
(e)



(f)

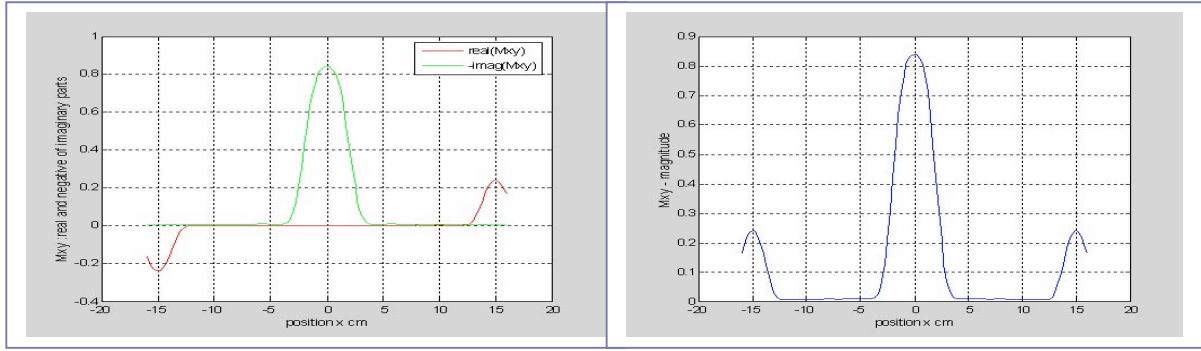


(g)



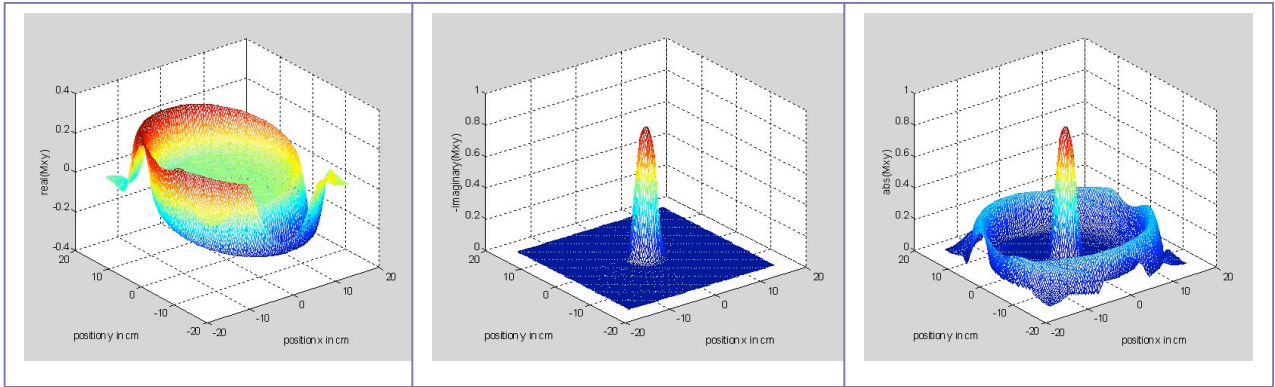
(h)

Fig 15. Excitation profiles using uniform k space weighting (a) real parts and negative of imaginary parts of the 1D excitation profile. (b) absolute values of the 1D excitation profile. (c)(f) real parts of the 2D excitation profile (M_{xy}). (d)(g) negative of the imaginary parts of the 2D excitation profile (M_{xy}). (e)(h) absolute values of the 2D excitation profile (M_{xy}).



(a)

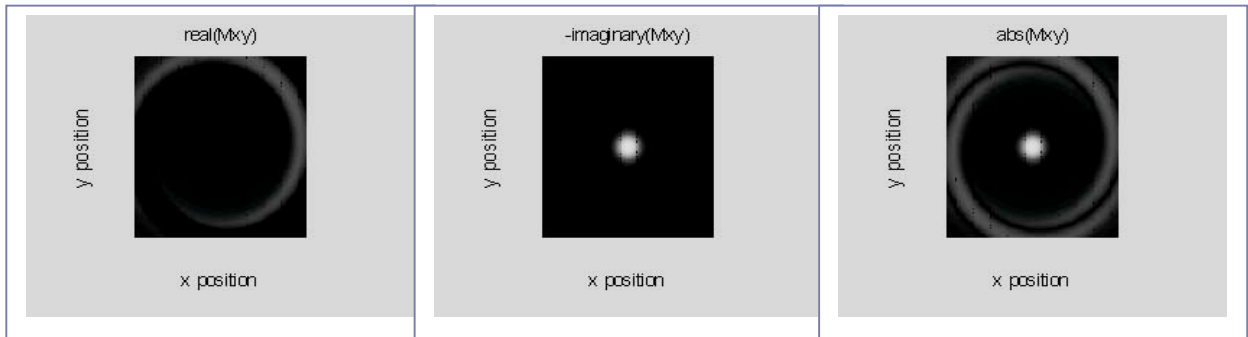
(b)



(c)

(d)

(e)



(f)

(g)

(h)

Fig 16. Excitation profiles using windowed jinc k space weighting (a) real parts and negative of imaginary parts of the 1D excitation profile. (b) absolute values of the 1D excitation profile. (c)(f) real parts of the 2D excitation profile (M_{xy}). (d)(g) negative of the imaginary parts of the 2D excitation profile (M_{xy}). (e)(h) absolute values of the 2D excitation profile (M_{xy}).

From fig (15) and fig (16) we can see that in the case of uniform weighting function a much smaller region than for what it was designed is excited. Whereas by using windowed jinc weighting function a circle of approximately 4 cm (as designed) is excited. Uniform weighting also results in ringing (evident from the 1D profile). Hence, we can conclude that windowed jinc k space weighting gives better results than uniform k space weighting.

Shifting the excitation profile:

All the excitation profiles discussed above were at the isocenter. We can shift the excitation profile by modulating the RF pulse $B1(\tau)$. From equation (1) we know that

$$M_{xy}(r, t) = iM_0 \int_{-\infty}^t \gamma B1(\tau) \exp(i2\pi k(\tau, t) \cdot r) d\tau$$

Modulating $B1(\tau)$ we obtain,

$$B1m(\tau) = B1(\tau) \exp(-i2\pi k(\tau, t) \cdot r_0)$$

Substituting $B1(\tau)$ by $B1m(\tau)$ in equation (1) we get the transverse magnetization ($M_{xym}(r, t)$) as,

$$M_{xym}(r, t) = M_{xy}(r - r_0, t)$$

From the above equation we see that by modulating the RF pulse by $\exp(-2\pi k(\tau, t) \cdot r_0)$ the excitation profile shifts by r_0 . Hence when r_0 is $2+0j$ the excitation profile shifts by 2 cms in the x direction (fig(17)(b)) and when r_0 is $0+2j$ the excitation profile shifts by 2cms in the y direction (fig(17)(d)).

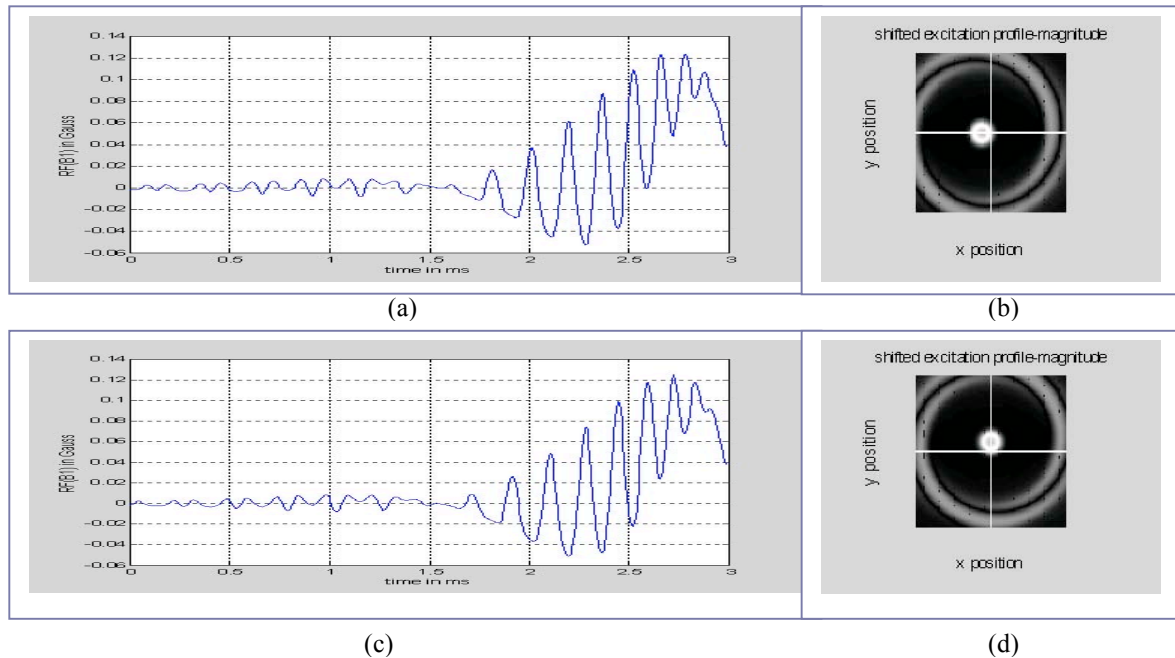


Fig 17. (a) and (b) shows the real part of the modulated RF pulse and the corresponding 2D excitation profile resp. for a shift of 2 cm in the x-direction. (c) and (d) shows the real part of the modulated RF pulse and the corresponding 2D excitation profile resp. for a shift of 2 cm in the y-direction. The trajectory used had $N=8$ turns, $SBW=8$, $FOV=16\text{cm}$, $\Delta r=1\text{cm}$ and windowed jinc weighting was used.

Effect of Gradient Delays on the excitation profile:

The time lag between the instant when the gradient is expected to begin and when it actually begins is defined as gradient delay.

1. the excitation profile with a constant angular rate spiral:

Equation for constant angular rate spiral as defined before is

$$kr(t) = k \max(1 - t/T) \exp(i2\pi N(1 - t/T))$$

If the gradient delay is δ then the equation of the spiral changes to

$$kr(t - \delta) = k \max(1 - (t - \delta)/T) \exp(i2\pi N(1 - (t - \delta)/T))$$

For small values of δ ,

$$kr(t - \delta) = k \max(1 - t/T) \exp(i2\pi N(1 - (t - \delta)/T))$$

Hence,

$$kr(t - \delta) = kr(t) \exp(i2\pi N\delta / T)$$

This shows that gradient delay of δ leads to the rotation of the spiral trajectory by $2\pi N\delta / T$ and hence the rotation of the excitation profile about the isocenter (fig 18(a)).

2. the excitation profile with a variable angular rate spiral:

A spiral with variable angular rate sampling is given by,

$$kr(\tau(t)) = k_{\max} (1 - \tau(t)/T) \exp(i2\pi N(1 - \tau(t)/T))$$

Where, $\tau(t)$ is the angular rate at which the spiral is traversed.

For a small gradient delay of δ ,

$$kr(\tau(t - \delta)) = kr(\tau(t)) \exp(i2\pi N\delta\tau'(t)/T)$$

Thus, in the case of variable angular rate spirals each spatial frequency is rotated by a different amount (fig 18 (b)).

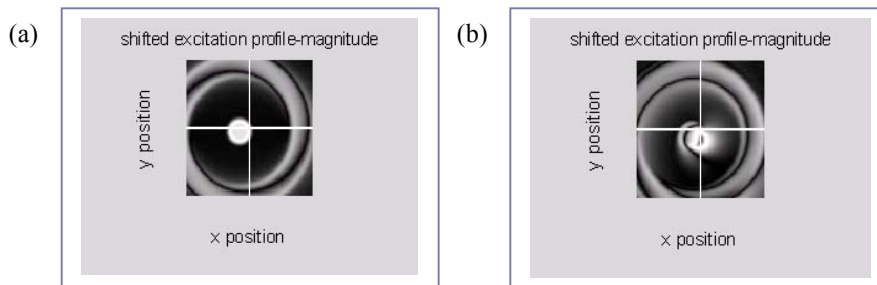


Fig 18. (a) Excitation profile with a gradient delay of 15microsecs and a shift of 4cm along x for a constant angular rate spiral. (b) Excitation profile with a gradient delay of 15microsec and a shift of 4cm along x for a variable angular rate spiral. The spiral trajectory used had N= 12 turns, SBW=8,FOV=24cm and

Effect of Off resonance on the excitation profile:

Off resonance can occur due to various reasons such as main field inhomogeneity, susceptibility difference, chemical shift etc.

From equation (5) we have,

$$W(k(t)) = \frac{\gamma B1(t)}{|\gamma G(t)|}$$

If $B1(t)$ is applied at an off resonant frequency “ ω ”, then the actual weighting $Wa(k(t))$ is,

$$Wa(k(t)) = \frac{\gamma B1(t)}{|\gamma G(t)|} \exp(-i\omega t)$$

Substituting the $W(k(t))$ by $Wa(k(t))$ we compute the excitation profile for off resonance condition. It is seen from fig (19) (a) and fig (19) (b) that off resonance blurs the excitation profile. It is also evident that greater the off resonance greater is the blur.

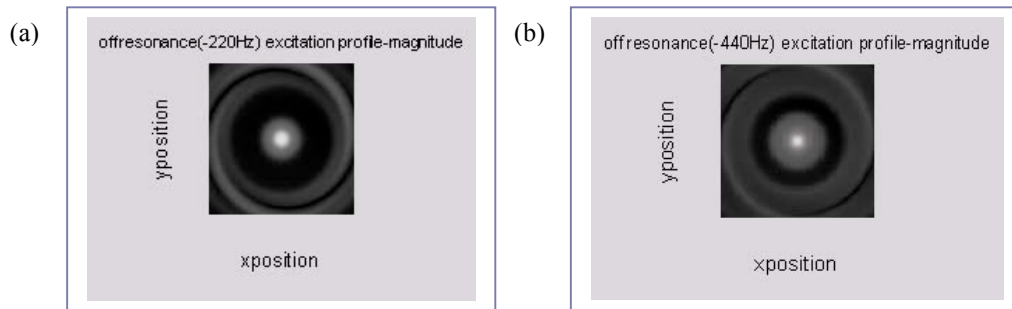


Fig 19. (a) excitation profile with an off resonance of -220Hz. (b) excitation profile with an off resonance of -440Hz. The spiral trajectory used had N=12 turns, SBW=8, FOV=24cm and uniform k space weighting was used.

Variable Density Spirals [3]:

The gradients define the k space trajectory and the excitation profile by depositing RF energy at discrete locations in the k space. Generally the trajectories traverse the k space with equal resolution. However, these methods generate aliasing signals outside the excitation field of view. In variable density spirals the central region of the k space (low frequencies) is sampled with higher resolution. That is, greater RF energy is deposited in these regions. Variable density spirals remove unwanted signals from outside the excitation field of view. In this project we compare the excitation profiles obtained using uniform density spiral (N= 12 turns, resolution=1cm, kmax=0.5 cycles/cm) and those obtained using variable density spiral (N=12 for $0.5 > k > 0.25$, N increasing linearly from 18 to 24 ($N=12t+12$) for $0.25 > k > 0$, kmax=0.5 cycles/cm). As expected, from fig

20, we see that the unwanted signals outside the excitation field of view are reduced by using variable density spiral.

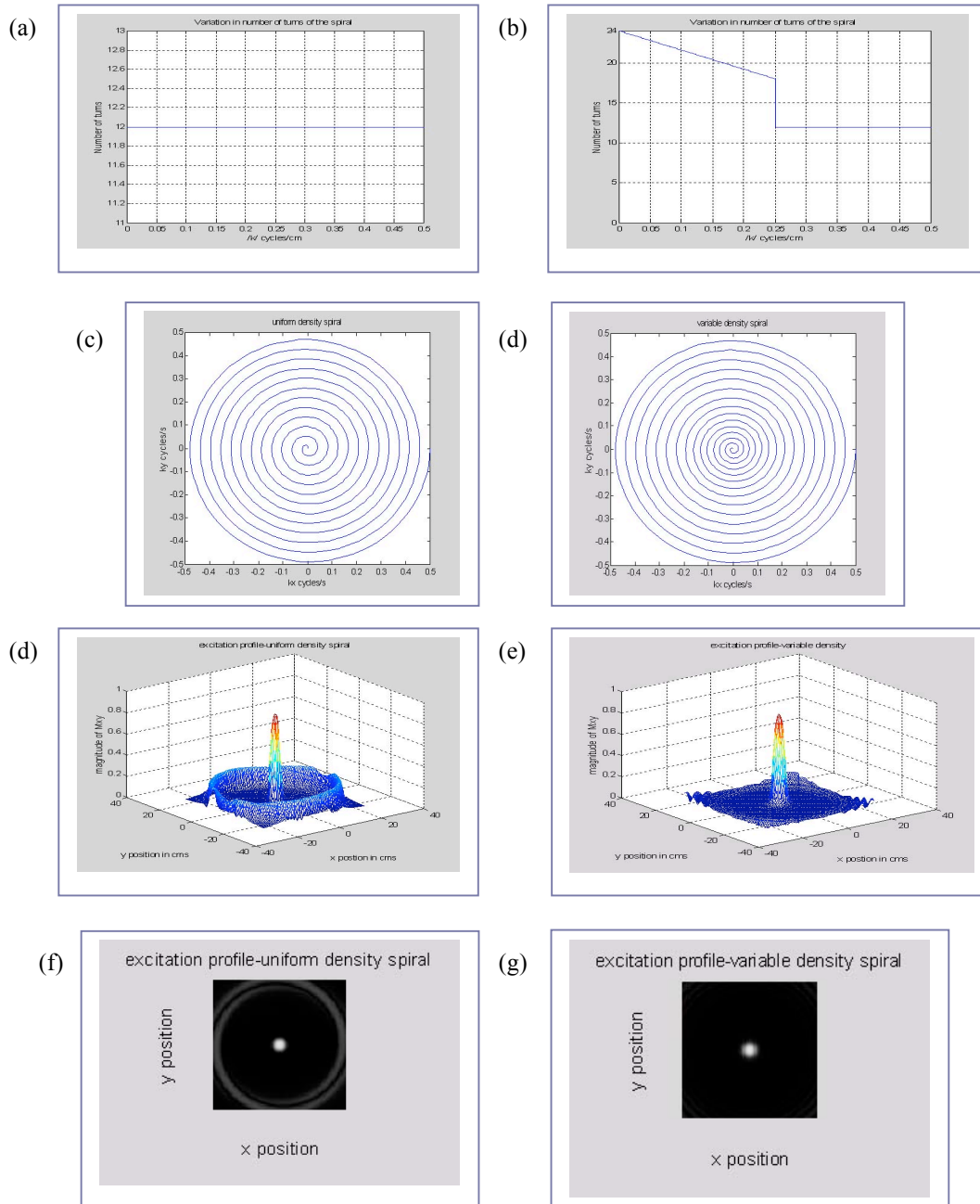


Fig 20. (a), (c), (d) and (f) shows the plot of # of turns verses $/K/$ cycles/cm, spiral trajectory and the 2D excitation profiles resp. for a uniform density spiral trajectory. (b), (d),(e) and (g) shows the plot of # of turns verses $/K/$ cycles/cm, spiral trajectory and 2D excitation profiles resp. for a variable density spiral trajectory.

References

- [1] Principles of Magnetic Resonance Imaging by Dwight G. Nishimura
- [2] <http://www.stanford.edu/class/ee469b/>
- [3] Christoph Schroder, PhD, Peter Bornert, PhD, and Bernd Aldefeld, PhD. Spatial Excitation using Density Spiral Trajectories. Journal of Magnetic Resonance Imaging 18:136-141 (2003)